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## FORCE AND TORQUE.*

BY HENRY CREW,

*Northwestern University, Evanston, Ill.*

When your secretary courteously extended me an invitation to say something on this occasion, I was not inclined, either by taste or by superabundance of time and energy, to accept; yet I find myself always unable to forego an opportunity to advocate what appears to be the simplest point of view in scientific thought.

One thing is certain; you will not expect me to say anything new on a subject so old as dynamics. But there are other facts of importance besides those recently discovered; and sometimes very old truths need rediscovery. So important, indeed, are clear views on elementary mechanics that even if I make not a single suggestion of value to any one of you, I shall nevertheless feel justified in directing attention to a matter so vital in the pursuit of physical science.

The time is not very far past—if past it is at all—when the chief aim in the science of physics was to explain all physical phenomena in terms of mechanics. But I believe there is now a consensus of opinion, that what physical science seeks above all else is unity of view. Whether this unitary conception be dynamical or electrical, or something neither dynamical nor electrical, is a matter of secondary importance. The main search is after objective unity in our system of experience.

But even if unity should be secured by reducing ordinary inertia to an electrical phenomenon, this will not in the slightest degree affect the commanding position which dynamics now occupies in the science of physics.

In this dominant branch of natural philosophy, there is one idea about which, more and more, the entire subject of mechanics has come to center itself, namely, the idea of *force*.

*A lecture delivered before the C. A. S. & M. T., November 27, 1908.

Before this assembly, one need hardly recall that the "fundamental" quantities, so-called—length, mass, and time—are fundamental only by the sense that standards of these three quantities are at present more conveniently prepared and preserved than those of any other three.

If in the future it should become more practicable to employ another set of fundamental units, as *length, force,* and *time* are now used by the engineer, the change can be made without modifying a single dynamical law. The choice of these three units from which all others shall be derived is then a mere bit of opportunism and depends in no way upon anything fundamental in their nature.

Quite otherwise is it with the idea of force, the one idea which is involved in practically all the laws of mechanics, the one idea in terms of which nearly all mechanical explanations are conceived, the one idea whose introduction into modern thought is synonymous with the creation of the science of mechanics.

In emphasizing the fact that force has been the fundamental idea for two hundred and seventy years, and is likely to remain so for a long while, I am not unmindful of the brilliant attempt made by Hertz, at the very close of his too short life, to substitute for force a set of mechanical constraints (sometimes visible, sometimes invisible) from which by application of Gauss' theorem of Least Constraint, all the phenomena of nature may be reduced. Nor do I forget various attempts to make energy assume the present position of force in the foundation of dynamics. But I think it may be fairly said that none of these new systems have yet proved their superiority to the classical mechanics. At the same time, we must frankly admit that this may be merely another way of saying that the time is not yet ripe for them. And this admission we make all the more cheerfully, since our present system of mechanics which employs forces most freely, uses them, as Hertz suggests, only as idle pulleys. It remains yet to be shown that a system which does *not* contain these idle pulleys will work smoothly.

Unfortunately this term force is one of the most abused in the entire English vocabulary. The twenty-three widely different definitions of it contained in the great Oxford dictionary occupy no less than six columns of fine print.

My purpose is therefore, first of all, to direct your attention to the one single clear and definite sense in which the word

force is employed in physics. I was on the verge of saying *modern* physics, but when I recall the fact that we owe the idea to Galileo, that Newton, and Huygens, and D'Alembert, and Lagrange, and Helmholtz, and Kelvin have, in royal succession, employed the term in the same sense with Galileo, it becomes plain that the accurate use of the word is not confined to our own generation, or even to the last century, but covers the entire history of physics.

If certain instances of misusage do occur, it will be found either that they do not happen in the writings of men acknowledged as scientific authorities, or that as in the case of Lagrange, who is criticised by Hertz for defining force as a "tendency," the writer's words do not correspond with his practice.

Not infrequently does it happen that the clearest conception of an idea is obtained by going back to the master mind that first felt the need for that idea, and invented a term to connote it; in this case we go back to Galileo, who unmistakably laid the foundation of dynamics in his "Dialogue concerning two new sciences," published in 1638. He it was, who by the introduction of the experimental method, also made dynamics, from the very start, a branch of physics rather than of mathematics.

Previous to the time of the great Italian, men were familiar with the fact that certain bodies, such as "weights" and springs, produced certain pressures; and these pressures were detected by the muscular sense; or, as the Germans put it, the *Gegenstand* was recognized by its *Widerstand*.

But concerning the cause of this pressure and concerning the appropriate measure of this effect the predecessors of Galileo were completely in the dark. Whether, for instance, the pressure of a body held in the hand was determinative of the distance through which this body would fall in a given time or of the velocity it would acquire in so falling, or of the acceleration it would exhibit, was by no means clear, previous to the publication of the *Dialogues*. At this point Galileo made three great strides:

(1) He showed that in the case of any single heavy body, this quantity, which had hitherto been thought of as pressure, really determined the acceleration of the body when it fell.

If a body were held on a steep inclined plane where the pull along the plane was great, then the acceleration along the plane was also great when the body was released. If the slope of the

plane was slight, the pressure along the plane was slight, and the acceleration was slight. A body when supported vertically exerted its maximum pressure and experienced its maximum acceleration when released. Into velocity and distance entered other factors so that neither speed acquired nor space fallen were proportional to the pressure alone.

(2) But, secondly, Galileo found by experiment that the acceleration of gravity is the same for all bodies regardless of their size; from which fact he must have seen that the change of pressure which the hand experiences when a one pound weight is replaced by a ten pound piece is not therefore due to a change in *acceleration*. Thus without ever reaching the conception of mass as measured by inertia, he did prove that the vague something hitherto called pressure depends upon two separate factors —and that one of these is acceleration.

(3) The third of Galileo's achievements is a confession of ignorance. It was clear to him that friction was unavoidable, that all motions were in fact accelerated, and that the cause of this acceleration was in no case understood. Very well then, said he, let us confess our ignorance, let us preserve a becoming modesty, and say that whenever a body changes its velocity, we do not in general understand the cause, but *in our thought*, we will substitute for the cause something which we shall call "force," and this force we shall measure by the acceleration which it produces.

*Force is therefore, merely a fictitious something with which we agree to replace an unknowable cause when we observe a body under acceleration.*

This, as I conceive it, is the unique sense in which *force* has always been used by the physicist.

In the *Dialogues* it is Salviati, the character who represents Galileo himself, that is made to say, "It is in vain that we search after the causes of natural motions, concerning which each philosopher entertains a view which is different from that of every other philosopher." Newton at the very outset of the *Principia* repeats this idea when he says, "For I here design to give a mathematical notion of those forces without considering their physical causes and seats." * * * Wherefore the reader is not to imagine that by these words, I anywhere take upon me to define the kind of manner of any action, the causes or the physical reasons thereof.

The ancient dictum that "science is the search for causes and philosophy for the cause of causes" would therefore hardly meet the approval of either Galileo or Newton. Indeed, so far from considering force as a cause, Galileo would rather regard it as an effect, produced by unknown causes, but measured by the acceleration of the body. Science avoids causation, seeks classification, desires unity.

By confessing that the question of causation lies beyond the range of physics, Galileo delimited the subject in a manner which has enabled it to make tremendous strides. It is precisely by throwing aside, so far as possible, all metaphysical considerations that the physicist has been able to make trustworthy predictions and that the engineer has become a prophet.

The importance of a clear definition of force is not likely to be overestimated. The lack of such a definition is well illustrated by the history of the idea of "vital force," an idea which for want of definite description has never borne any fruit. The belief that there is something more involved in physiology than mere chemistry and physics is all but universal; just what this "something" is, however, and just how it may be appropriately described or measured, no one yet appears to know. Until this step is made, "vital force" will probably remain an idea too general and indefinite to be useful.

**Mass.**

The definition of force was completed and perfected when Huygens and Newton discovered the variability of gravity. Newton keenly appreciated the meaning of this discovery and on the first page of his *Principia* assigns the name of "Mass" to that factor of weight which does not vary when the acceleration of gravity changes—the factor which remains constant even when a body is removed from the earth to the orbit of the moon.

**Newton's Third Law.**

One other achievement of Newton completed the foundation of modern dynamics. Even before the birth of this great Englishman, Galileo had thoroughly grasped the Second Law of Motion, and had frequently employed it in the theory of projectiles. More than that, Galileo had correctly conceived the First Law of Motion, expressing it as merely a special case of the Second Law. However, the notion that there are always two forces involved in every dynamical transaction, had not dawned upon the Italian pioneer.

It remained for Newton to establish, and to describe as his

Third Law, the general statement that whether the action between two bodies results in acceleration or merely in a stress, two equal and oppositely directed forces are always involved.

In the case in which this interaction is a stress, i. e., in the case of mutually equilibrating forces without acceleration, there is no ambiguity or difficulty.

**Poggendorff's Experiment.**

When, however, the only action of a force is to produce acceleration, the reaction is not always so patent. The following experiment, which is described by Poggendorff in *Ann. der Physik, 92,* 179 (1854), and reproduced in Mach's *Mechanics,* exhibits the mass-reaction of an accelerated body in the most convincing manner. (*Experiment shown.*)

**The Role of Analogy.**

Pursuing the same subject, namely, the simplest point of view in these matters, let us pass for a moment to the dynamics of rotation. For here I venture to think, there has been an immense amount of time and energy needlessly wasted over problems which, when solved by translational methods, are exceedingly complex and which in the light of analogy are exceedingly simple.

When one recalls that in modern science any phenomenon is said to be explained only when it has been pigeon-holed with other phenomena which we already understand, it is not a little surprising that in the teaching of mechanics the use of analogy is so seldom employed. These ideas I wish to illustrate by the explanation of three fundamental phenomena.

**The Gyroscope.**

The first of these is the behavior of a rotating wheel when its axle is supported at one end only.

(*Experiment with bicycle wheel shown.*)

To make this matter clear, one has only to bear in mind that mass in translation is the strict analogue of moment of inertia in rotation; that linear displacement corresponds rigidly to angular displacement, and that the one remaining fundamental unit, time, is identical in both.

Then just as in the case of a particle moving with uniform circular motion, the centripetal force—the force which produces this motion—is at right angles to the direction of momentum, $mv$, is at right angles also to the axis about which the direction of momentum is rotating with an angular speed $w$, and is numerically equal to the vector product of these two.

$$F=[mv\ w],$$

so also, in the case of the bicycle wheel, the torque which produces precession is at right angles to the axis of the wheel's angular momentum, I Ω, at right angles also to the axis of precession, and numerically equal to their vector product

$$L=[I\ \Omega\ w]$$

In like manner, the centrifugal force which may be considered as the reaction of the centripetal is strictly an analogue of the torque produced by a precessing wheel to which may be considered as the reaction of the torque required to produce precession. Their numerical values are those of F and L, given above, with their signs changed.

**Schlick's Ship Steadying Device.**

The next illustration which I wish to consider is one identical with the bicycle wheel, except that here we shall consider the work done, while there we considered the torque exerted.

(*Experiment with rolling-ship model shown.*)

Let this square iron frame represent a transverse section of a vessel which is rolling, not pitching, in the sea. The vibration of the attached pendulum produces the roll of the vessel. A heavy wheel is placed amidships and is set spinning about a vertical axis.

Parallel to the keel of the ship is attached a rod which connects the lower end of the oscillating frame of the spinning fly-wheel with a dash-pot or friction-brake.

The angular momentum of the heavy fly wheel I Ω, is about a vertical axis; the precession. *w*, is produced by the rolling, and is about a horizontal axis parallel to the keel.

It follows, therefore, after the analogy of centrifugal force, that the torque [I Ω *w*], exerted by the fly wheel is about an axis athwart-ship.

Now anything which opposes this torque will oppose the rolling of the ship; and if this torque is allowed to do work, this work will be at the expense of the energy of roll of the ship. Schlick therefore introduces a dash-pot filled, I believe, with water and glycerine to absorb the work done by this athwart-ship torque.

The same expression for moment of force, I Ω *w*, may be derived from translational dynamics; but the route is longer, and, for most minds, vastly more difficult.

To show that the mere exertion of torque, without absorption

of energy will not diminish the roll, we have only to substitute, as I shall now do, for the dash-pot a spring which will return to the ship during *one* half the roll, practically all of the energy imparted to the spring during the *other* half of the roll. (*Experiment shown.*)

**Brennan's Monorail Car.**

A third illustration of the simplicity of view-point obtained by use of rigid analogy is furnished by the monorail car recently devised by Mr. Brennan in England.

This machine consists essentially of an iron car-frame upon which is mounted a horizontal, fore-and-aft axis. This fore-and-aft axis in turn carries a pair of vertical frames in each of which a heavy fly wheel is kept spinning about a horizontal axis.

These frames with their fly wheels have three distinguishing features:

(1) The entire load—frame, wheels, and all—is mounted on the fore-and-aft axis so that it can rotate freely, yet with its center of gravity in the axis, so that, of itself, it has no tendency to roll.

(2) These two wheels are maintained spinning, in opposite senses, about the axes transverse to their respective frames.

(3) These two frames can each precess about a vertical axis; but they are geared together in such a way that the precession of one is equal in amount but opposite in sense to that of the other. The result is that one wheel counterbalances the effect of the other except about a fore-and-aft (longitudinal) axis; and here they combined their effects. If now the car frame tends to roll about the single track as an axis (i. e. to upset) only one of two things can happen:

(a) The car frame may strike against the projecting axle of one of the fly wheels, which will not only prevent the car from upsetting farther, but will hurry on the precession by rolling friction of the shaft in such a way as to completely right the car.

(b) Or it may happen that the car frame will strike a projection on one of the wheel frames, by which the car will be prevented from farther upsetting and by which also precession will be caused, such that the live axle will a little later be thrown into action and the car righted exactly as in the first case.

Each side of the car frame is provided with the same kind of stops or lugs, so that whichever way the car starts to roll, it is at once prevented from rolling farther; and not only so, but the torque exerted by the car on the gyroscopic wheels is such as to hurry on the precession and to roll the car back until its center of gravity lies vertically over the track.

(*Monorail car shown.*)

Here again the righting torque of each fly wheel is measured by $I \Omega w$ where I is the moment of inertia, $\Omega$ the angular velocity of each wheel, and $w$ the rate at which the wheel frame is precessing.

This expression is identical with that found in the case of the bicycle wheel, and in the ship-steadying device, and is identical in form with $mv w$, the ordinary expression for centrifugal force.

* * *

While most of what I have said is both old and obvious, I trust nevertheless, that I have brought from the background into the foreground the three following ideas:

(1) Galileo's definition of force, presenting it not as a cause of motion, but as a measurable circumstance of actual motion.

(2) Newton's idea of mass and mass-reaction, and

(3) The power of exact analogue in dynamics.

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The Cambridge Botanical Supply Co., Cambridge, Mass., have just issued their 1909 catalogue of botanical supplies. It is an up-to-date and complete catalogue, containing everything the teacher of botany needs for laboratory or lecture room work. Descriptions and photographs of the many special devices of their own make are given. There is also a very complete list of microscopical and lantern slides. The special feature of this issue is the listing of Perret's volcanology slides, for which they have the exclusive agency for this country and Canada. Fifty-six slides, made from actual photographs, showing every phase of volcanic eruptions, are listed. Every teacher of geology and physiography should be interested in these slides.

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Chemistry teachers will be interested in an article entitled "Making Poison by the Ton," which appeared in the *Technical World Magazine* for January, 1909. It furnishes a good illustration of the use of the process of sublimation. The ore, a quartz rock, contains the arsenic combined with iron and sulphur, and by roasting the arsenic oxide is formed and this condenses in brick flues four feet wide, six feet high, and three hundred yards long. These flues terminate in tall stacks, but even this long trap does not prevent the escape of some arsenic to wreak vengence on the vegetation of the surrounding country. One smelter near Everett, Wash., produces and ships to New York thirty-five tons of this poison each week.

## CALORIMETRY MADE CONCRETE.

By F. M. Gregg,

*State Normal School, Peru, Neb.*

In dealing with problems in calorimetry in a high school where apparatus was limited, the writer devised a set of transportation problems which were taken up before the subject of calorimetry was considered and served so well in the after clearing up of that subject that he ventures to present the device for the benefit of the readers of School Science and Mathematics.

These problems were introduced as a simple diversion from the regular physics lessons, and nothing was said about any after application. Indeed, the student was allowed to make his own discovery and application later. Generally but one lesson was given to the transportation problems. To begin with, a rough sketch of the scene that heads this article was put upon the blackboard, the conditions of transportation explained, and the problems then given out.

In the solution of the problems it is understood that a traveler in passing toward A from D, or any intermediate point, *pays* fare as follows: the distance from D to S is 1,000 miles and the fare on the airship going toward S is ¼c per mile; to be transferred from the airship to the steamboat costs $5.37 per passenger; the distance from S to Z is 100 miles, and to go any distance on the steamboat toward Z costs 1c per mile; it costs 80c to be transferred from the steamboat to the sledge, and the fare by sledge anywhere between Z toward S is ½c per mile, the total possible distance being 273 miles.

It is further to be understood that in this strange country the traveler *receives* pay for traveling from A or intermediate points toward or to D just what he would have to *pay out* if he were to go his same journey in the opposite direction. Typical problems follow.

Problem 1.—What would be the total cost of a trip from D to A? What would one receive for going from A to D?

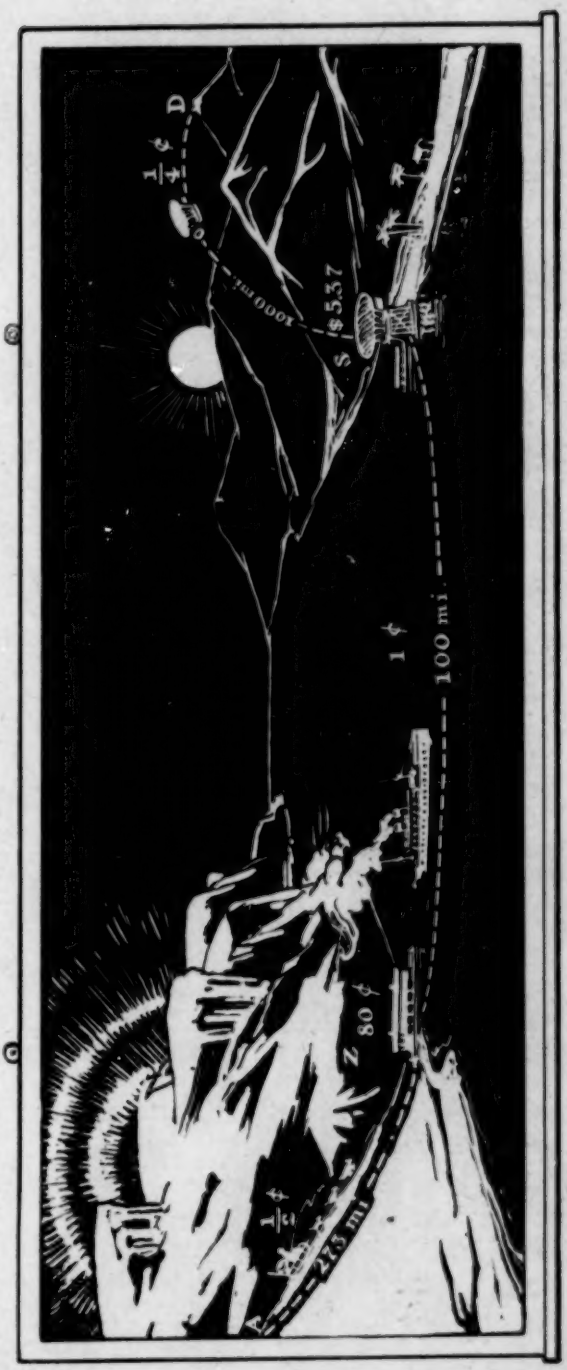
Problem 2.—What would one receive for going from A to S but without being transferred to the airship? How does this sum compare with the cost of being transferred from the airship to the steamboat?

PROBLEM 3. —What would five passengers receive in going from a point 20 miles to the left of Z to a point 20 miles to the right of Z?

PROBLEM 4. —How much would it cost one to be transferred from the airship to the steamboat, conveyed to Z, and be transferred from the boat to the sledge?

PROBLEM 5. —A passenger leaves a point 20 miles to the right of S and goes to a point 58 miles to the left of S. How many passengers would have to leave a point 56 miles to the left of Z and go to meet the one passenger and receive for it just what the one passenger had paid out?

PROBLEM 6. —Five passen-



gers leave a point 112 miles to the right of S and go to meet, somewhere between S and Z, twenty-five passengers coming from a point 10 miles to the left of Z. How many miles from the right of Z must they meet so that the one party spends just as much as the other party receives?

## COOLING THROUGH A CHANGE OF STATE.

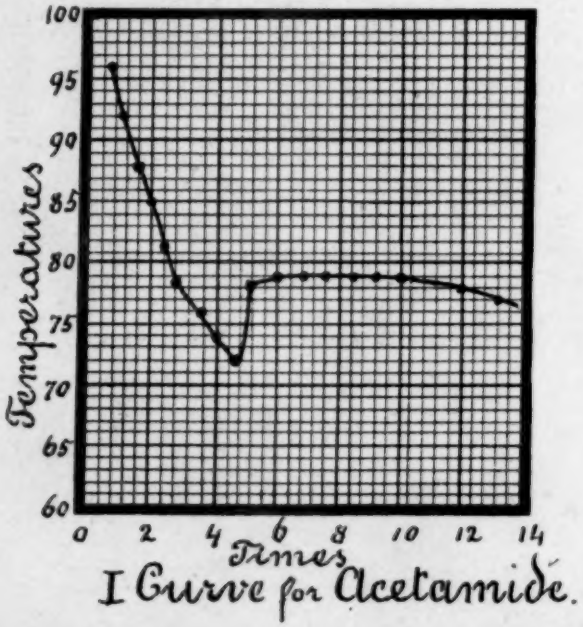
By C. K. Studley,

*State Normal School, Chico, Cal.*

This experiment is neither new nor original but follows the same lines as Exp. 21 in Millikan & Gale's Lab. Course in Physics, but so far as I know the material used is new in this capacity, and is so simple and inexpensive that it ought to be of use to teachers of physics. The experiment referred to above calls for acetamide which is neither very cheap nor easy to obtain,

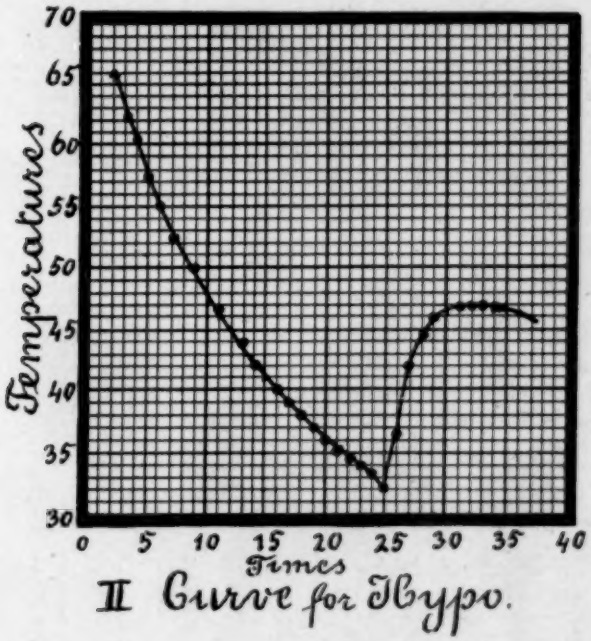
The apparatus necessary for this experiment is a thin walled test tube, a bunsen burner or alcohol lamp, a test tube holder, a thermometer and some sodium thiosulphate or ordinary hypo such as is used in fixing photos.

Fill the test tube nearly full of the crystals of hypo and carefully melt the crystals by holding over the gas jet or alcohol




I Curve for Acetamide.

lamp. If they are heated carefully the temperature need never rise above 70° C. or 75° C. As soon as the crystals are all melted, insert the thermometer. Let it stand about half a minute and take the temperature. Take readings every minute, watching the liquid carefully. In an ordinary room it may be cooled to about 30° C. or possibly a few degrees lower, but as soon as there ceases to be a marked change in the temperature drop in a fresh crystal of hypo and note the results, at the same time continuing to take readings. The accompanying curve was



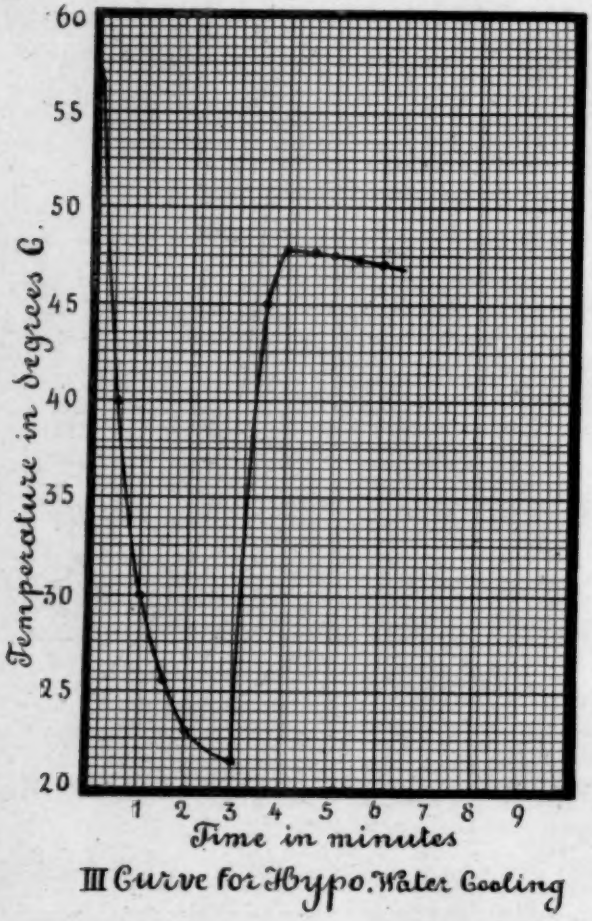

II Curve for Hypo.

made in this way. Longer intervals, say two or even three minutes, could be taken if desired. For the sake of comparison I have reproduced the curve of cooling given for the acetamide in order that the advantages of the hypo may be readily seen.

If it is desirable to shorten the experiment, a relative curve may be made by putting the test tube into a dish of cold water as soon as the first reading is taken and taking readings every half minute. As soon as it is cooled to about 22° C. or 23° C. remove it from the water and drop in a crystal of hypo. This will show the heat of crystallization more strikingly than by the other method and the time consumed by the entire experiment will be less than one fourth what it was in the other case. Below (Fig. 3) I give a curve obtained in this way.

Comparing the two curves 1 and 2 we will see that in the case of the acetamide as soon as crystallization begins the thermometer begins to rise and finally indicates a change of 7° C.. while in the case of the hypo there is a like rise of 15° C. Take a glance at Curve 3 and it will be seen that there is a corresponding rise of 25° C.

Hypo is not expensive and can be obtained with little difficulty so it seems that this alone ought to commend it to the teachers of physics.




III Curve for Hypo. Water Cooling

## THE USE OF THE MICROSCOPE IN THE PHYSICAL LABORATORY.

By E. J. Rendtorff,

*Lake Forest Academy, Lake Forest, Ill.*

When most physics teachers were adopted to their present positions they inherited, amongst other paraphernalia, the inevitable large air pump, the static machine of Newtonian design and a plethora of microscopes of divers species. I was perhaps exceptionally unfortunate, as eight of them fell to my lot.

It is difficult to sell second hand microscopes at a fair price, or to trade them for more desirable apparatus in less pretentious cases. The question therefore arises: What shall we do with them?

I wish to offer a few suggestions that may be of some slight service to another fellow sufferer.

A. The micrometer microscope.

On a glass disc that will fit loosely in the tube of the ocular, have a scale ruled of about 100 lines to the centimeter. Move the diaphragm D, Fig. 1, in the ocular tube until the scale S is seen sharp and distinct, when the upper lens is attached.

Place the ocular in position and focus the microscope on a glass millimeter scale, or if the microscope is a very powerful one, on a scale having at least one millimeter division graduated into ten parts. This focusing must, of course, be done so that no parallax exists between the micrometer scale S and the image of the millimeter scale placed on the stage.

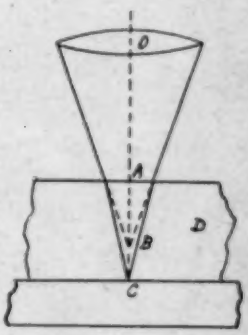



Fig. 1.

Let $N_1$ divisions of the micrometer correspond to $N_2$ of the lower $^1/_{10}$ mm. divisions. The value of one of the micrometer divisions is therefore

$$\frac{N_2}{10N_1}\text{ mm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

This space will not be a constant for one particular ocular, as it depends also upon the objective lens employed.

To measure any small distance, such as the diameter of a fine glass capillary tube, or hair, the grating space of a diffrac-

tion grating, etc., it is only necessary to focus properly and then determine the number of micrometer spaces, $N_3$, that they cover.

The desired distance is then $N_3 \frac{N_2}{10N_1}$ mm.....(2)

If the space on the reference scale is 1 mm., and $N_1$ the number of micrometer divisions per mm. of the magnified scale, the thickness of the body $= \frac{N_3}{N_1}$ mm.....(3)

B. Newton's rings.

On the stage of the microscope place a Newton's rings apparatus consisting of a slightly curved glass resting on a plane piece and mounted in a frame with adjusting screws so that the shape of the interference rings can be altered at will. Focus the microscope and adjust the pressure screws until the rings are circular.

When monochromatic light, such as that from a Na flame, is projected from the lower mirror the bands will be alternately black and yellow. In some cases 100 or more concentric rings can be produced.

If R denotes the radius of curvature of the curved glass, $r$ the radius of a black interference band and $\lambda$ the wave length of Na light, then for any band viewed with transmitted light at normal incidence:

$$\left.\begin{aligned} r_1 &= \sqrt{\tfrac{1}{2} R\lambda} \\ r_2 &= \sqrt{\tfrac{3}{2} R\lambda} \\ r_3 &= \sqrt{\tfrac{5}{2} R\lambda} \end{aligned}\right\} \text{ etc. .....(4)}$$

If the rings are viewed with reflected light incident at an angle of practically 90°:

$$\left.\begin{aligned} r_1 &= \sqrt{R\lambda} \\ r_2 &= \sqrt{2R\lambda} \\ r_3 &= \sqrt{3R\lambda} \end{aligned}\right\} \text{ etc. .....(5)}$$

In case the radius of curvature, R, be determined by some other method, the wave length of the Na light can be calculated.

The radii of the interference bands are, of course, measured with the micrometer described in the previous experiment.

C. Index of refraction.

A. Of glass.

Focus the microscope on a fine scratch, C, Fig. 2. Place the glass, D, whose index of refraction is desired, in position as illustrated and refocus on the scratch. Measure the distance BC, through which the objective lens, O, must be elevated for this adjustment. With a micrometer or vernier calipers now measure AC, the thickness of the glass.

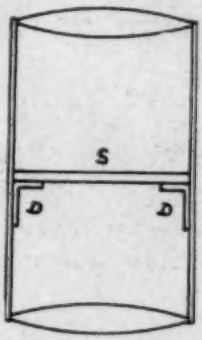



Fig. 2.

The index of refraction =

$$\frac{AC}{AC - BC} \quad \ldots\ldots\ldots\ldots (6)$$

If the glass is quite thick the ocular and objective must be chosen to give a low magnification, but the most powerful combination possible should always be selected.

B. Of liquids.

Focus the microscope on a fine scratch on the upper surface of a glass trough having a flat bottom. Partially fill the trough with the liquid whose index is desired and refocus as before. Finally focus on the top of the liquid, which can be done with ease if a few fine dust particles are allowed to float on the surface. Measure the various elevations of the objective and substitute in Eq. (6).

A microscope with a divided head for determining the elevation is especially adapted to this experiment.

D. The microscope as a polariscope.

On replacing the ocular by a mounted Nicol's prism, removing the objective and placing another Nicol under the stage of the microscope a very efficient and desirable polariscope results. In place of the lower Nicol a plate of glass held at the polarizing angle can be substituted for the reflecting mirror and fair results obtained. The latter method is possibly the more instructive.

The simpler polarization experiments, such as the color of mica films and wedges, the configuration in uni—and biaxial crystals and unannealed glass, quarter wave plates, ordinary and extraordinary rays in Iceland spar, etc., are extremely fascinating, though not understood by the elementary student. They, however, have a cultural, or rather an aesthetic value.

## MAKING A CHEMICAL COLLECTION.

By Wilhelm Segerblom,

*The Phillips Exeter Academy, Exeter, N. H.*

It has been the privilege of the writer during the past ten years to formulate and carry out definite plans for a large, comprehensive, and systematically arranged collection of substances that are of interest to students of chemistry. Since it has recently come to his notice that several other teachers of chemistry in secondary schools are establishing chemical collections, a few words may not be out of place in regard to the experience of the writer in this school where the conditions have been rather favorable for the work and where several schemes have been developed with gratifying results. Our collection at present numbers about seventeen hundred specimens, and it is hoped that it will eventually reach twenty-five hundred specimens, this latter number being considered a reasonable limit to the amount of illustrative material which can be used to advantage in the department of chemistry in a secondary school.

For convenience the collection is divided into three parts: (1) the elements, (2) the compounds, and (3) specimens to illustrate the chemical arts and manufactures. The first two divisions are mainly of theoretical interest, because the elements and compounds are arranged according to their scientific classification. Much, however, of practical value and of technical interest is interwoven, for the substances stand side by side on the shelves as they are made in the laboratory, as they occur in nature, and as they are used in technical processes. The third division of the collection is of practical interest primarily, showing how the substances classified in the other two divisions are used in the various industries. This division naturally covers a very wide field. Already several firms have donated specimens to illustrate the manufacture of their special lines of goods. These samples often include specimens of the raw material, specimens of material during the different stages in the process of manufacture, side products, residues, and finished products.

For further convenience this article will consider the following topics: cases, bottles, labels, classification, sources of material, gases, minerals and head pieces; these will be followed by a brief consideration of the separate groups of elements and compounds, giving such results of our experience as may possibly be helpful to others engaged in this line of work.

## Cases.

The most convenient form of exhibition case for the elements and compounds is one at least seven feet and eight inches high, outside measurement. The base need not be over six inches high, and for the molded top seven inches may be allowed, though of course this will depend upon the style of molding used. This leaves about six feet and a half for the height of the door, that is, for available exhibition space. Cupboards and drawers are omitted entirely because all the specimens should, of course, be constantly exposed to view. The door sashes and parting beads should be cut down to the smallest possible dimensions consistent with strength and stability. Much can be gained in this respect by attention to small matters; for instance, if the doors are hung so that each parting bead carries only one pair of butts, these beads may be narrowed appreciably. The case should present the maximum of glass surface and the minimum of surrounding wood frames. We have found that a door six feet and six inches high by twenty-six and a half inches wide, both outside dimensions, will take four lights of glass, each twenty-four by eighteen inches; this being a trade size of glass, no cutting is required. A suitable depth for the case is eleven inches outside; this will leave about nine inches for the width of the shelf. The case should contain eight shelves, the base being called the first one. Alternate shelves should be the full depth of the case, and these shelves should come directly behind the cross-bars of the sashes. The other shelves should be about five inches and a half in depth and should come about midway between the other shelves. This arrangement exposes all specimens to full view without obstruction from cross-bars and so on. The advantage of this style of case over a deep case containing narrow shelves arranged as a flight of steps is that all the specimens are brought much nearer the glass door and it is therefore much more convenient to examine them; furthermore, these shelves are particularly well adapted for both the minerals and the two sizes of bottles which will be mentioned below. The cases may be made with any number of doors, from two to twenty, according to the size of the space allotted to the cases. The doors should be supplied with good, non-corroding locks, and one key should fit all locks.

The cases for the third division of the collection may well be, for consistency's sake, of the same dimensions as those just described, but the depth may very properly be doubled.

The cases should be built of some wood, such as white pine, that will not warp or check after standing some months. The outside may be finished to correspond with the rest of the finish in the room. The inside should be painted; pure white is not a desirable color on account of the large number of white salts that the collection will eventually contain; some tint should be chosen that will not be conspicuous in itself but should form a good background for not only the white salts but also the variously colored minerals. The writer has found a light drab, dull finish, a satisfactory color.

Some of the cases, preferably those containing the first two divisions of the collection, should be placed in the hall and in as good a light as possible. This brings the collection permanently before the students of the department and increases their interest as well as the value of the collection itself for reference.

### Bottles.

All chemicals should be exhibited in bottles of uniform size, but these should be of two sizes, four-ounce and twelve-ounce. They should be made of a good grade of glass, absolutely free from color and as free from bubble imperfections as possible. Wide-mouth, glass-stoppered bottles can be used throughout, for liquids as well as for solids. The bottles should be bought by the gross, in order to ensure uniformity. We were fortunate in getting a bottle blown in an iron mold; this left a horizontal mold mark just below the neck, and this mark was exceedingly helpful, because when the tops of the labels were put even with this mark the labels were brought to a uniform height above the shelf.

### Labels.

For use on the bottles a plain white label two and a half inches by one and three eighths inches is a good size. This may have a double light-rule border near the edge to improve its appearance, but as much free space as possible should be left for the writing. The name of the school and the words "CHEMICAL COLLECTION" should be printed near the top of the label. For use with minerals and other unbottled specimens a white, stiff card, three and three quarters inches by two and one eighth inches should be used. This should be bordered and printed to correspond with the sticker labels. For supporting these cards beside specimens when it is not desirable to lay the cards on the shelf beside the specimens, Dennison's ticket pins and card holders are exceedingly convenient. It is often helpful to have a large label card to indicate the different subdivisions of the collection. These cards may be as large as three by five inches if desired, and should, of course, be printed to correspond with the smaller labels. The ruled border, however, may be a little heavier.

It should be remembered that fully-described and well-written labels enhance the value of a collection immensely and that many valuable specimens may be comparatively worthless for

collection purposes if not accompanied by proper labels. The curator of the collection should therefore see to it that all specimens be accompanied by proper labels. He should even suggest to prospective donors the advisability of supplying additional label information. Some one has said that a museum is a collection of descriptive labels illustrated by suitable specimens. The spirit of this definition might well be the underlying principle governing the development of a secondary school collection.

### Classification.

The elements may be subdivided into metals and non-metals. The compounds may be divided into oxides, hydroxides, hydrides, chlorides, bromides, iodides, fluorides, cyanogen compounds, sulphides, carbonates, sulphates, silicates, oxalates, chromates, nitrates, phosphates, and miscellaneous compounds. The specimens in the arts and manufactures division will naturally vary to some extent with the locality and with the interest that manufacturers may show in the collection. But they should consist primarily of specimens illustrating those industries which are most clearly dependent upon chemical reactions.

### Sources of Material.

If the funds of the school allow it many substances may be obtained by direct purchase. In nearly every school the stock-room contains a supply large enough to allow samples to be transferred from the stock-room to the collection. Many substances can be made by the students themselves from stock on hand, and this work can be arranged either as regular advanced work or as voluntary work for the better scholars. In all cases, substances so made should have on the label the name of the student who made them. This device will always bring about fresh additions from other students. Many manufacturers are ready and glad to donate samples of their products to such collections, but for obvious reasons it is not advisable to mention the names of any such firms in this article.

Lastly, of course, friends of the school may become interested enough in the collection to donate specimens or money for their purchase. It should be emphasized here that the instructor should have some well-formed plan in mind toward which he is constantly working; this will enable him to recommend judiciously in case of prospective donations.

## Gases.

The gaseous elements, oxygen, hydrogen, nitrogen, and chlorine, may be prepared in the laboratory, purified if necessary, dried and stored in sealed glass bulbs. The writer has used for this purpose an inexpensive modification of gas-collecting tube; this modification consists of a thin glass bulb about three inches in diameter with two small bore glass tubes about six inches long fused into opposite sides of the bulb; these are very convenient because the straight tubes may be bent at right angles close to the bulb, and after the bulb has been filled with gas the ends of the glass tubes can be drawn off in the blast lamp and hermetically sealed. The main advantage of bending the tubes is that the bulbs will then lie in any position in which they are placed and there is no danger of their rolling around. A small label with the symbol of the gas should be pasted upon one of the straight tubes and a larger card label with complete information should be laid beside the bulb. Bromine, both in liquid and in vapor form, may well be similarly shown.

Of the gaseous compounds, ammonia gas, phosphine, hydrogen sulphide, hydrochloric acid gas, three oxides of nitrogen, two oxides of carbon, and two oxides of sulphur, may be exhibited in glass bulbs. In preparing sulphur trioxide the bulb should be immersed in a freezing mixture and the sulphur trioxide gas passed into it for some time, care being taken, of course, to have all apparatus perfectly dry; when enough of the snow-like oxide has collected, the ends of the glass tubes should be drawn off while the bulb is still in the freezing mixture; after standing several weeks the amorphous sulphur trioxide vaporizes and condenses on the side of the glass in long, beautiful, needle crystals, sometimes two inches long. For collecting carbon monoxide gas the bulb described above cannot be used because the gas ignites spontaneously and explosively during the drawing off of the glass tubes; instead, a separatory funnel with stem cut off close to the lower stop-cock may be used.

In this connection it is of interest to note that sulphur dioxide and nitrogen tetroxide may both be liquified at ordinary temperature by passing the gases into the bulbs packed in freezing mixture; the bulbs might well be sealed before being removed from the freezing mixture.

## Minerals.

Minerals have a distinct place in such a collection as is being

here described, because so many chemical compounds appear as minerals and it is desirable to show these compounds in as many different forms as possible. If the school has practically no minerals already on hand, by far the best beginning is to purchase from a reliable dealer a collection of a hundred and seventy-five to two hundred specimens especially prepared for high schools and academies. Such a collection has usually been made up with considerable care, and though it illustrates only the common or important species and varieties, these come from widely-distributed localities. Such a collection is much more representative than any collection of minerals that the secondary school teacher has time or opportunity to assemble personally. Specimens in such a prepared collection are usually well labeled with mineral name, variety name, chemical name, formula, and locality. Since these specimens are for cabinet use instead of for individual study, they should be of pretty good size, say from four to five inches long and from three to four inches wide. Such a collection of minerals makes a substantial beginning for a chemical collection, and as other minerals are desired subsequently they can be obtained separately. A smaller mineral collection than the one just described would not be consistent with the size of the chemical collection as outlined, and a larger one would not give students and other interested friends an opportunity to donate additional specimens. It goes without saying that extremely rare minerals and specimens showing but slight variations in the same mineral are out of place in a secondary school collection. It should constantly be borne in mind that the collection is not to confuse the mind of the student by multiplicity of specimens, but rather to leave sharp impressions of the groups of compounds and the more characteristic and common specimens within each group.

### Head Pieces.

The cases themselves, particularly if they are large, receive a finishing touch if a very large specimen is placed on top of each case. Substances that serve this purpose well are large masses of gas-retort carbon, say about two feet long; large cross sections of petrified wood, showing the complete trunk from center to bark; a huge mass of gypsum and a large lump of commercial fibrous salammoniac, such as is used in iron industries. Other specimens will suggest themselves to the teacher.

(*To be continued.*)

## SOME ROMAN BUILDING MATERIALS.

By Nicholas Knight,

*Cornell College, Mount Vernon, Ia.*

Long ages after the far-famed "seven hills" were lifted above the sea by volcanic action, and all the region round about the "eternal city" had become dry land, there came an era of large fresh water lakes and a river Tiber many times larger than the present stream. These fresh waters were heavily impregnated with carbonic acid, and they dissolved quantities of the limestone with which they freely came in contact. This limestone was deposited in various places and is known as *travertine*. The extensive deposits near Tivoli, about twenty miles from the city, at the foot of the Sabine hills, have furnished a large amount of building material. The quarries there located have been utilized from the time of the emperors to the present. The Colosseum is built of travertine, and it has defied the vicissitudes of time during sixty generations of men. This serves to indicate the wonderful staying qualities of the material. The beautiful red sandstone in the British houses of Parliament erected in about 1830 is rapidly becoming disintegrated, although in a comparison of this kind it must be remembered that the English climate like the American makes more strenuous demands upon a building than the milder, dryer climate of sunny Italy.

The durability of travertine is still recognized and the new palace of justice, which has been several years in building and which when completed will rank as one of the finest modern structures in all Italy, is being constructed of this material.

We have never noticed an analysis of the rock although many have doubtless been published. A specimen analyzed as follows:

| | |
|---|---|
| $CaCo_3$ | 90.50% |
| $MgCo_3$ | 3.65% |
| $SiO_2$ | 3.36% |
| $Fe_2O_3$ | 0.76% |
| $Al_2O_3$ | 2.06% |
| | 100.33% |

The chemical analysis scarcely furnishes a clue to its enduring qualities which are rather to be sought in its physical characters. It *looks* durable. It is vesicular, the vesicles lined with

drusy calcite and light yellowish gray in color. It contains some long threadlike cavities, apparently of organic origin. It is crystalline and an old surface contains no grains that are detachable by the finger. The specific gravity is apparently higher than that of ordinary limestone.

Rome was fortunate in the variety of its native construction materials and in its easy accessibility to the marble quarries of Asia, Africa, Greece and the islands of the Mediterranean. Augustus proudly boasted that he found Rome brick and left it marble. What a pity that so many of the finest buildings and works of art, some of the world's greatest masterpieces, were ruthlessly burned for quicklime! Fragments of these ancient marbles recently picked up on the Forum (when the custodians were not looking) gave the following results when analyzed:

| | |
|---|---|
| $CaCo_3$ | 90.37 |
| $MgCo_3$ | 2.47 |
| $SiO_2$ | 3.17 |
| $Fe_2O_3$ | 2.46 |
| $Al_2O_3$ | 1.37 |
| | 99.80 |

There is quite a striking resemblance in the composition of the two materials which one would scarcely expect.

Our thanks are due Mr. Irvin B. Bleeker of the Cornell College chemical laboratory for making the foregoing analyses.

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The quantity of coal consumed in the manufacture of coke in 1907 was 61,946,109 short tons, valued at $72,784,851. The value of the coke produced from this coal was $11,539,126, a difference of $38,754,275, which represents the profits on the coking operations less the cost of manufacturing and the expenses of administration and selling. In 1906 the value of the coal used was $62,232,524, and the value of the coke produced was $91,608,034, a difference to cover all expenses of manufacture, administration, and profits of $29,375,510.

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Consul A. G. Seyfert of Owen Sound, Ontario, furnishes the following statistics concerning cement in Canada:

One-half of all the cement manufactured in Canada is manufactured in the locality of Owen Sound, the daily output here being 3,000 barrels. The demand for concrete material for building the transcontinental railroad has increased the output to the full capacity of the plants during the past year. During 1907 Canada produced 2,500,000 barrels of Portland cement, and imported 1,000,000 more, nearly all from the United States.

## A PROFILE RELIEF MAP.

By Percy E. Rowell,

*San Rafael High School, San Rafael, Cal.*

While having a class in physical geography make profiles from a contour map, the idea of making a relief map by means of a series of profiles occurred to me. As preliminary practice for the class, I drew several ideal contour maps, about eight inches square, with contour lines rather far apart but representing a rise of one hundred feet. I chose an arbitrary vertical scale of ¼″=100′, which makes the work very easy for beginners. These ideal maps represented the most common forms of the earth, and I tried to incorporate the various combinations of contour lines. The profiles were run ¼″ apart, and when cut out were separated by strips of wood about ¼″ thick (allowing for the thickness of the cardboard). I used the unglazed four-ply Bristol board, which is one mm. thick. The spaces between the profiles were then filled with moulding wax, and the relief map was complete.

The pupils evinced so great an interest in the work that I then attempted the enlargement of the United States Geological Survey topographic sheet of our vicinity, by means of enlarged profiles. The result was so gratifying that I have been urged to give the benefit of my experience to others who may desire to do the same kind of work.

The class was divided into pairs and it was the duty of one of each pair to read the map and of the other to plot the reading. The pairs exchanged their duties as they desired. Profiles were run every millimeter. No one pair ran successive profiles, but skipped as many profiles as there were pairs. Thus, if there were fifteen pairs the profiles for any one pair would be fifteen millimeters apart. The advantage of this method is that each pair is a check upon the two adjacent pairs, the mistakes in the successive profiles being self-evident. Since it was desired to increase the size of the map twenty-five times, the distances, as read between successive contour lines, were multiplied by five, before plotting, and the adjacent profiles were separated five millimeters between centers.

The method of procedure is rather hard to describe without drawings, but is as follows: My class comprised seven pairs.

Each pair marked its copy of the map every seven mm. on both sides, one pair taking divisions 0-7-14-21-28, etc., the second pair taking numbers 1-8-15-22-29, etc., the third pair marking divisions 2-9-16-23-30, etc., and so on, the numbers given being the number of millimeters above the bottom of the map. To work upward was found easier than to work downward. Then the reader of each pair placed a millimeter rule on the map, where it was marked, and read the elevation on the left side of the map, at the very edge. This reading was plotted by his partner in a manner which will be shown later. Then the reader, using a magnifying glass (2″ diameter is large enough), read the number of millimeters on his scale at which the next hundred contour line came; readings were made to half millimeters. He multiplied this number of millimeters by five (we made tables to prevent careless errors), and gave the result, together with the altitude, to his partner, who plotted it. The horizontal distance in millimeters was given first, followed by the altitude in hundreds of feet. This process continued across the map and then the exact altitude of the edge was given and plotted.

Each pupil who plotted was provided with a plotting board, which was designed for this purpose. It consisted of a piece of ⅞″ planed redwood, 8″ wide and one meter long, with a meter stick fastened near the edge but on the flat side of the board. This edge was countersunk so that the thickness of the meter stick, which projected above the surface of the board, was equal to the thickness of the Bristol board plus the thickness of a celluloid 60° drawing triangle. Thus the top surface of the triangle and the top surface of the meter stick were flush. The one plotting would take a piece of Bristol board as long as the width of the finished map, and wide enough for the particular profile, plus 4″ for a purpose to be shown later, and fasten it, by means of thumb tacks, flush with the right hand end of the board, at which end the numbering on the meter stick begins. Then he was ready to receive the first reading.

Since the map had to have some thickness, even at sea level, two inches were chosen. This accounts for two inches of the extra four inches. Therefore, if the reading was at sea level, a dot would be placed two inches above the bottom of the cardboard. No vertical measurements were necessary, however, since a scale was placed on the under side of the transparent triangle.

Having the scale placed on the under side of the triangle avoided parallax. This scale began, that is had its zero, two inches above the base of the triangle and it was divided into arbitrary divisions, numbered with successive hundreds. Since the contour interval was twenty-five feet, these divisions were subdivided into quarters.

When the plotter received the reading, he would slide the triangle along to the proper horizontal distance and place a dot at the desired hundred mark, close to the vertical edge of the triangle. Therefore, there was no measuring and the dots were placed exactly over the proper position on account of the squareness of the triangle. All horizontal readings were given as total distances from the edge of the map so that there could be no additive errors. When all the readings were plotted a smooth curve was run through all the dots and the profile was cut out.

The duties of the reader were not confined merely to the reading of horizontal distances and elevations. He had to note every road, brook, shore line, and marsh which his profile was crossing, and this was plotted, not on the profile but on the other two inches of cardboard, above the profile. When this part was cut from the profile it became a template, containing all the information concerning the country which it crossed. Both profiles and templates were numbered and signed by each pair.

As fast as the profiles were finished they were assembled, being separated five millimeters between centers by means of strips of wood. Since the thickness of the wood varied, paper strips were added from time to time, always keeping the profiles in their proper places. All errors were apparent, and when these had been corrected the map was nailed together, profile after profile. In order to obtain squareness and rigidity, a table was provided with a false top, on which were firmly fastened two pieces of wood, 2″x2″, longer than the two dimensions of the map, and at right angles to each other. After a few profiles had been nailed together, screws were run through one of these pieces into them. All profiles were slid up against the other piece and then nailed to the growing map. As the map was nailed together there was a tendency to buckling, which was prevented by running long screws from the under side of the false top up into the assembled map.

When all the profiles were assembled, the spaces between them were filled with a soft modeling wax and the surface of the map moulded, using the profiles as guides. The map was finished with a smooth surface, obtained by careful rubbing with the fingers. Then it was placed in a wooden flask, made of two-inch planks tightly fitted around it, and the flask was filled with plaster of Paris of a creamy consistency. This produced a matrix which was easily removed from the modeled map, since the modeling wax was oily. After the matrix was dry, little irregularities were smoothed out, the surface well smeared with vaseline, and a cast made. The same flask was used, the cast taking the position formerly occupied by the original map, but, of course, bottom side up.

After becoming thoroughly dried the cast was smoothed with fine sandpaper and painted with the conventional colors. In this work the templates were used to obtain the location of all points. The template number, multiplied by five, gave the distance in centimeters of the template's position above the starting point, or bottom of the map. The locating of points and the painting of the map were the hardest part of the whole work and were the most tedious.

If relief maps are to be made the same size as the topographic sheets from which the information is obtained, the system of piling up cut-out contours is the best method. If the relief map is to be larger than the topographic sheet, the method of enlarged profiles is much better than the use of the pantograph to enlarge contour lines. Mistakes are not magnified and there is a sure check upon the work.

The scale of the finished map is $1''=1{,}000'$, horizontally, and $1''=500'$, vertically. The map has been framed with $\frac{7}{8}''$ Oregon pine, $7''$ wide, which is fastened into strips of wood which had been cast into the plaster of Paris, of course below the surface of the map. This map will be on exhibition at the Alaska-Yukon Exposition, at Seattle.

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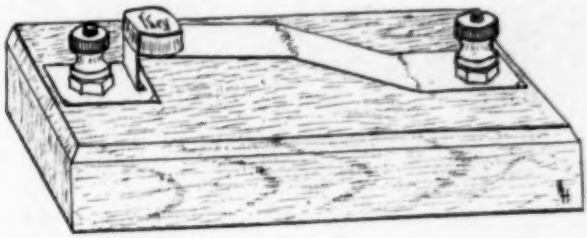
**Nearly 2,000,000 h. p. in the form of gas is allowed to escape from the blast furnaces of the country. This condition is rapidly being changed by the installation of gas engines to develop the power. There is also a great fuel waste in the manufacture of coke, beside the loss of many valuable by-products. It is estimated that these losses amount to $50,000,000 annually.**

## APPARATUS YOU MAY NEED.

By E. L. Harrington,

*Rock Port, Mo.*

When I took charge of the laboratory here I found neither an electric key, nor a commutator, and I feel safe in saying that there are many high school laboratories none too well supplied with them. As I would need all the money the board could spare me to purchase apparatus I could not make, I decided to make my commutators, keys, and considerable other apparatus.

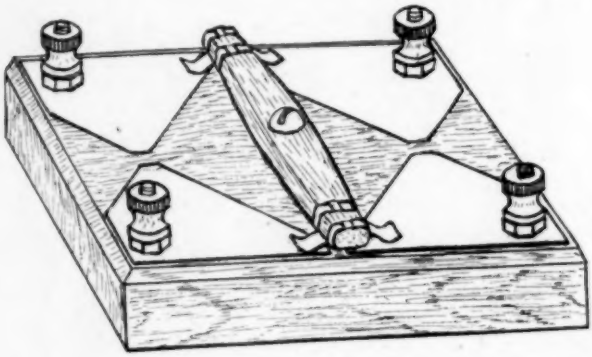


The drawing shows the kind of keys constructed. From a sheet of spring brass was cut a strip 12 cm. long and 1.2 cm. wide. About 2.5 cm. of this was cut off for a contact plate, while the rest was bent as shown in drawing. Battery bolts were used for the binding posts, and the corners of the spring and plate near these were bent downward and pressed into the wood to prevent turning. Lacking the ebonite buttons found on most electric keys, it was found necessary to cut a strip two mm. wide from each side of the vertical portion of the key spring. These were bent backward, and on them, and the top of the key spring a button of sealing wax was molded. These keys were found to be quite satisfactory.

The inconvenience and often the expense connected with the use of the old-fashioned mercurial commutators by an ordinary high school pupil, led me to devise a new kind—simple in construction, substantial, always ready for use, and of sufficient accuracy for elementary use.

The four brass plates of the commutator were made by cutting along the diagonals of a brass sheet about 6.5 cm. square. About 4 mm. of each of the inner corners were pressed vertically into the wood base, while the outer corners were cut off to fit the base which was 7 cm. by 11 cm. in size. Battery bolts

were used for binding posts. The turn key was made 6 cm. long out of a piece of wood not easily split. For each of the end bridges I used a piece of brass spring 2.7 cm. by 1.5 cm. each end of which was divided into three parts, the outer and narrower portions being clamped over the end of and in notches cut in the turnkey. By tightening the center screw, the contacts were made as firm as desired. Enough of these commu tators were made to supply the class, and they were found to be satisfactory in use. Their action is instantaneous and they may be used for a key and a commutator at the same time.



By a visit to the rubbish pile of the city telephone office I secured a supply of horseshoe magnets, generators, electro-magnets, parts of telephones, battery bolts, and a large quantity of insulated copper wire of various sizes. From these things were made galvanometer coils, galvanoscopes of three windings, resistance spools, which I mounted substantially after the class had wound them to various resistances, Wheatstone bridges, a contracting helix, etc. High resistance coils, so expensive to buy, may be made by carefully mounting on wood bases the electro-magnets of telephone ringers. Through such methods about $125 worth of apparatus was secured for our high school laboratory with practically no cost. All apparatus made was carefully constructed, painted and polished, so that it would present an attractive appearance. And I found that by keeping the apparatus as though it were on display in the cases, the scholars were more careful and interested in the laboratory, and especially the school board became interested in and generous in the support of the laboratory.

# A KEY TO THE COMMON WINTER TREES ABOUT MILWAUKEE.

I. N. MITCHELL,

*Milwaukee State Normal School.*

Editor's note: Although the title of this paper indicates a purely local significance, it is of pretty general significance, first, in that the trees covered by the synopsis are common throughout many of the north central states, and secondly, in that the method of work used by the author of the paper should prove of great assistance to secondary teachers of botany.

—*O. W. C.*

The following key is the expression of a desire to help students to help themselves in the field work. It does not pretend to be technical, nor to appeal to the systematist, as such. It includes, with one exception, only the common trees to be found in parks and streets of Milwaukee.

The sycamore is included because of its very characteristic and noticeable exfoliation of the bark. The only term used not readily understood or ascertainable, is the word "smocking."

This term is borrowed from the dressmaker, who catches the plaits of a waist together alternately, thereby causing the surface to appear with diamond shaped cavities or reticulations. The white ash is perhaps the best illustration of the application of the term.

In section one, "cone bearing trees" means trees actually bearing cones, though the student is very likely to know that the evergreens of this part of the world are cone bearing whether the cones are present or not. Similarly, in the first part of section two, the tamarack is given because, though technically a cone bearing tree, the cones are not always present.

## Section I.—CONE BEARING TREES.

* *Evergreen trees and shrubs.*

1. *Leaves needle shaped.*

A. In pairs.

a. Needles longer than the finger, tree. Austrian pine, Pinus Austriaca 5

*Note*—The numbers following the names of trees refer to pages of a Tree Study Bulletin in which descriptions of the trees are given. The bulletin may be had on request. Milwaukee Normal School Bulletin, Vol. 2, No. 4.

b. Needles shorter than the finger, tree. Scotch pine, Pinus sylvestris 7

c. Needles short, one inch long, spreading shrub. Gray or northern scrub pine. P. Banksiana

B. Needles in fives.

a. Needles slender, soft, whitish below. White pine, Pinus strobus

C. Needles single, scattered, one-half to three-fourths inch long.

a. Needles one-half inch, stiff, very dark green, tree, cone-shaped, branches horizontal, Christmas tree. Black spruce, Picea nigra

b Needles about ¾ in., dark green, branches drooping. Norway spruce, Picea excelsa 7

c. Needles about ¾ in., slender, pale green. White spruce, Picea alba

d. Needles about ¾ in., stiff, numerous, crowded, distinct blue-green. Colorado blue spruce, Picea parryana 8

2. *Leaves flat, single, usually* whitened below. Bole of tree with scattered blisters exuding balsam if pricked. Balsam fir, Abies balsamea 8

3. *Leaves minute, scale-like,* closely shingled on flat branches, aromatic when crushed. American Arbor vitae, Thuja occidentalis 9

4. Leaves very small, awl-shaped, spreading, stiff and sharp, very unpleasant to handle when dry. Red cedar, Juniperus Virginiana

** *Deciduous cone bearing trees.*

A. Bole short, head conical, branches whorled.

a. Head broad at base in proportion to height; cones an inch or more long. European Larch, Larix Europea

b. Head narrow at base in proportion to height; cones ½ to ¾ inch long. American larch, tamarack, Larix Americana 7

**Section II.—NOT CONE BEARING, I. E., NO CONES PRESENT. DECIDUOUS.**

* *Stem excurrent or appearing so.*

a. Branches whorled, horizontal.

b. Head broad at base in proportion to height.

European larch, Larix Europea

b. Head narrow at base in proportion to height.

American larch, tamarack, larix Americana 7

a. Branches alternate, ascending, long, beset with short, stout, peg-like branches about ½ inch long.

Ginkgo tree, Salisburia adiantifolia 5

a. Branches alternate, ascending, almost appressed, bark of branches olive or tan.

b. Head narrow candle-flame shape.

c. Bark of bole dark, rough.

Lombardy poplar, Populus dilatata 10

c. Bark of bole sage green, rather smooth.

Siberian poplar

b. Head wide candle-flame shape.

c. Bark of bole furrowed, light streaks or cracks in bottom of furrows.

Carolina poplar or cottonwood 10

** *Stem Deliquescent.*

A. Bark chalky white, peeling off in layers.

a. Branches stiff, self supporting.

White birch, Bitula alba 11

a. Branches weak, drooping.

Cut-leaved weeping birch. B. alba, variety pendula-laciniata 12

A. Bark dark, brownish gray, smoothish, branches stiff.

a. Lenticels conspicuous, short, black; new growth bristly hairy; spindle-shaped masses of red fruits often present at ends of branches.

Staghorn sumac, Rhus hirta 15

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*Note*—To determine the measure of branching, study the *outer ends* of the branches or the top of the tree.

a. Lenticels inconspicuous; branches stout, ascending, ending in large, varnished buds, small tree.
Horse chestnut (young), Aesculus hippocastaneum 17

A. Bark dark gray, smooth, marked with narrow, vertical, tan colored spots. Head round, thick.
Norway Maple (young), Acer platanoides 16

A. Bark dark gray, smooth, without markings; branches flat, i. e., branchlets all in one plane; buds mucilaginous.
Basswood, linden (young), Tilia Americana 17

A. Bark olive or olive brown, smooth, lenticels prominent; clusters of red berries often present; small tree.
Mountain ash, Sorbus aucuparia 15

A. Bark light gray and smooth throughout.

a. Trunk cylindrical; branches alternate, buds long, spindle-shaped.
American beech, Fagus Americana 12

a. Trunk cylindrical; branches opposite

b. Branches drooping, straggling.
Cut-leaved, silver or white maple.
Acer saccharinum, var. laciniata 16

b. Branches slightly drooping, usually conspicuously marked by numerous short twigs ending in clusters of buds giving the appearance of belt pins stuck along the branches.
(Young) Soft, white or silver maple.
Acer saccharinum 16

b. Branches not drooping, stiff, ascending.
Scarlet maple, Acer rubrum 16

a. Trunk subcylindrical (somewhat or nearly cylindrical) with conspicuous longitudinal ridges. Small tree.
Hornbeam, blue beech, water beech.
Carpinus Caroliniana 11

A. Bark chippy or scaly.

a. Very dark, chips one inch or more long,

concave, covering bole and reaching well along the branches.

Wild black cherry, Prunus serotina 15

a. Very dark below, light gray in the branches; chips large, flat, thin, peeling off.

(Old) Soft, white or silver maple Acer saccharinum 16

a. Very dark below, gray in the branches; chips firm, thick; branches stout, ascending, ending in large, varnished buds. (Old) Horsechestnut. Aesculus hippocastanium 17

a. Brown or red brown, rough below, smooth above; chips or scales large, thin, paper-like, deciduous, leaving smooth, olive patches.

Sycamore, Platanus occidentalis

a. Gray or brownish gray; scales long, narrow, loose, fibrous, running more or less spirally up the bole. Small tree. Iron-wood, Ostrya Virginiana 11

a. Gray, scales small, branches stout; some of the round-lobed leaves usually hang on through the winter.

American white oak. Quercus alba 14

a. Dark gray, scales small; light and smooth in branches; branches stout to the ends, opposite; bole somewhat smocked.

Black or water ash. Fraxinus niger 17

A. Bark very rough, ragged, shaggy, peeling off in long, flat, rough strips, rusty red on under side.

Shagbark hickory, Hicoria ovata 9

A. Bark rough, firm, conspicuously smocked.**

a. Smocking coarse, loose, elongated.

b. Head feather duster shape, branches thin; buds and bark not mucilaginous.

American or white elm. Ulmus Americana 14

Head same, branches stouter, stiffer, more numerous, buds and inner bark mucilaginous.

Red, slippery or large leaved elm, Ulmus fulva 14

b. Head flat, spreading, branches descending; bole usually shows grafting joint.

Camperdown weeping elm 14

a. Smocking coarse, loose, shallow; branches antler-like; leaf scars saucer shaped, conspicuous; often has long pods.

Catalpa, Indian bean, Catalpa speciosa 18

a. Smocking close, firm, regular.

b. Head open; bole gray, branches light, stout, doubly curved, ascending.

White ash. Fraxinus Americana 18

b. Head open, bole gray, branches dark, stout, drooping, usually shows grafting joint on bole.

European ash. Fraxinus excelsior 18

b. Head irregular, crowded with wiry branches, bole usually misshapen, warty, humpy, leaning, tree ugly in appearance, occasional exceptions, frequently carries its keys through the winter.

Box elder, acer negundo 16

b. Head round, symmetrical, full, branches continue the smocking in dark lines on a dark smooth bark.

(Old) Norway maple, acer platanoides 16

** Broken into deep, quite regular diamond shaped reticulations.

b. Head rounded; bole gray, smocking shallow.

Hard, rock or sugar maple, acer saccharum 15

b. Head ovoid; bole very dark, branches flat, i. e., branchlets (alternate) in one plane 17

(Old) Basswood, linden, Tilia Americana 17

A. Bark furrowed rather than smocked.

a. Furrows deep and wide, very dark.

b. Bole and limbs rugged, stout, dark, ridges of bole lighter.

Black oak, Quercus velutina 13

b. Bole rugged, low; limbs olive with many black scars; whole top of tree bent more or less toward the northeast.

White poplar, Populus alba 10

b. Bole rugged, tall, limbs tan, spreading.
Cottonwood (old) Carolina poplar.
Populus deltoidea 10

b. Bole rugged, low, branches thin and somewhat drooping, tan colored.
White willow, salix alba 11

b. Bole rugged, low, branches thin, stiff, yellow.
Yellow willow. S. alba. var, vittelina 11

A. Bark rough not distinctly chippy, smocked or furrowed.

b. Bole very dark; branches stout at base, opposite.
Red oak, Quercus rubra 13

b. Bole dark; branches stout, contorted, large limbs and upper bole conspicuously marked with narrow, shallow furrows and ridges that show as wavy lines.
Bitternut, Hicoria minima 9

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In U. S. Farmers Bulletin No. 333, Mr. W. A. Orton presents many most valuable suggestions upon Cotton Wilt. The nature and life habits of the wilt fungus is discussed, as well as its manner and rate of spread. It has been found that the time of planting and use of fertilizers exert considerable influence on the ability of cotton to withstand attacks of the wilt fungus. But in addition to these things it is possible to select varieties of cotton plants that are much more disease-resistent than others, and to care in seed selection cotton growers are asked to give much attention. It is believed that through proper selection and breeding most of the ills of the wilt disease may be eliminated. O. W. C.

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The *Experiment Station Record* states that appropriations of the last Congress to be administered by the Department of Agriculture aggregate $15,660,106.00. In addition to the lines of work already under way by the department, more extended attention is to be given to investigation of tuberculosis among cattle, standardizing the grading of cotton, food inspection, making of denatured alcohol under farm conditions. A permanent bison range was provided by instructing the President to reserve for that purpose 12,800 acres of the Flathead Indian Reservation in Montana. The American Bison Society has presented the government a herd of bison for this range.

Although the above total appropriation may seem large, it is to be remembered that it is but 1.5 per cent of the entire federal appropriation.
O. W. C.

## IV BOTANY.*

By John M. Coulter,

*The University of Chicago.*

Any statement of the problems of botanical instruction in secondary schools is necessarily only of temporary value. A few years ago the great problem was to get any kind of botanical instruction into the secondary schools; but this problem has in large measure disappeared. Then it became a problem how to introduce laboratory and field work into schools congested with studies and with pupils. This is no longer a problem in the sense that the claims of such work must be fought for. Thus the problems of today may cease to be problems presently, and will be replaced by others. The following statement, therefore, is merely a record of current conditions; and the purpose of this organization, as I understand it, is to change these conditions.

I. The Prepared Teacher.

The first great problem is the prepared teacher, a problem not peculiar to botany, but peculiarly conspicuous. It is the all-inclusive and perhaps the perennial problem. If it could be settled, there would be no other serious problems. In addition to teaching power, which is assumed to be the essential background, the prepared teacher of botany means several things.

(1) It means a general knowledge of the fundamentals of botany. There are three words in this statement I wish to emphasize. The preparation must be *general,* because plants are to be taught from several aspects, as their structure, their activities, their external relations, their use. To select any one of these aspects and to exclude the others, is to take the plant out of its natural setting. Nature presents itself as a composite thing; it is man who analyses nature for his own purposes. The preparation must include *knowledge,* as contrasted with information. Information may be obtained from reading, but knowledge comes only from experience, which in botany means wide contact with the plants themselves. No teacher is prepared, therefore, who has only a reading acquaintance with the subject. The preparation must include the *fundamentals,* because the facts of botany, like other facts, differ widely in importance; and the unprepared teacher is almost sure

*Read before the joint session of the American Federation of the Mathematical and Physical Sciences and Section I, of the American Association for the Advancement of Science.

to hit upon the trivial rather than the important. The dead level of facts is one of the blighting methods of scientific instruction, making out of the student a Gradgrind rather than a builder.

All of this means that no one should be allowed to teach botany who has not had laboratory and field experience with plants, in all their general aspects, under competent guidance.

(2) To some this preparation seems to be all that is necessary. It may be sufficient for college instruction, but not for instruction in secondary schools. The prepared teacher we are considering means also one who has a clear conception of the purpose of botany in secondary schools, as distinct from its purpose in higher institutions. To inject into these schools miniature duplicates of college and university courses is to defeat their purpose. The courses must be adapted to the age, the needs, the surroundings, the interests, the available time of pupils, as independently as if college courses did not exist. But unless this preparation of the teacher is cared for specifically the secondary schools inevitably will be infected by university laboratories in miniature. This means a certain amount of professional training, enough to make clear the peculiar function of the secondary school.

(3) The prepared teacher also means the ability to attack the subject in a variety of ways. There is no ideal method of first attack, for it may well vary, dependent upon many circumstances. Botany is like a great park, whose approaches are numerous. The most natural one to use is the one that happens to be most convenient at the time, the one nearest to the pupils. This means considerable grasp of material and great flexibility in presentation. A teacher who only knows one way is singularly handicapped. The principle just stated convinces me that it would be very unfortunate for any committee to assume to determine that some one method of approach is the best. This is to be determined by the competent teacher, whose special problem it is. It is certainly a waste of time to answer such a question for the incompetent teacher. The final application of power must be made by the teacher, and anything that encourages initiative in this respect is so much clear gain.

(4) The prepared teacher also means the disappearance of the well-worn excuse of difficulty in reference to any approach. Every approach is easy to him who knows the way; and to those who are under competent guidance every approach is

equally easy. I have used every group of plants and every aspect of plants as the initial point of attack, and I have found them all equally easy to pupils, and I have also found pupils equally ignorant of all of them. The excuse of difficulty always means the unprepared teacher. Botany is never difficult when taught by the prepared teacher; it is always difficult and futile when taught by one who is unprepared.

My conclusion in reference to this problem of the prepared teacher is that wherever unprepared teachers are at work botany will be in disfavor, and deservedly so. It has not so intrenched itself in the public mind as an educational necessity that it must be included in every curriculum in spite of poor teaching. Therefore, it is a serious question whether it should not be excluded from every school where it cannot be well taught. To prejudice the popular mind against a great and useful subject by mistreating it may perhaps be more dangerous than to remove it entirely from the danger of abuse. This is the old question whether it is better to teach a subject poorly than not to teach it at all; but it can hardly be so serious a question when in every school now there are numerous subjects which are well taught.

2. *Economic Botany.*

A current problem is the place of the economic aspects of botany in botanical instruction. The so-called practical side of plants is so interwoven with our whole civilization that it is evident to us now that it should not be neglected in any scheme of secondary education. It is now recognized to be a mistake ever to have constructed courses in secondary schools that did not include any of the economic aspects of plants, and that sometimes did not even include any useful plants. But this was a natural inheritance from the older educational theory, which dealt chiefly with mental discipline, and gave little or no thought to useful things. In fact, it was almost the pride of higher education that nothing useful was taught, and this sentiment naturally affected the secondary schools. In these days, however, there has been a swing to the other educational extreme; and the demand is growing for the so-called practical things. Botany is a subject peculiarly qualified to encounter this demand, and in some quarters it has taken the extreme form of a proposition to substitute agriculture for botany. The general motive underlying this proposition is commendable, for it voices a protest against the other extreme; but the change suggested is short-

sighted. It fails to recognize the fact that the most practical thing in the world is the foundation of pure science upon which applied science rests. To eliminate the fundamentals of botany in order to teach agriculture is to omit the underlying reasons for processes in order to teach empirical formulæ.

The ideal certainly lies somewhere between these two extremes. There must be a rational combination of pure and applied science. Perhaps the combination, expressed in the figure of a landscape, should be a background of pure science. against which its applications could stand out. It is true that in the main the fundamentals of botany can be obtained from useful plants, but there should never be a straining after such plants at the expense of a clear illustration of the principle to be established. For example, in presenting the vascular system of Monocotyledons, it would be natural to use one of the cereals; but it so happens that they are not at all representative of the situation, so why should they be used for this purpose? Just how to work out effectively the combination suggested, is one of the most important problems that confront us. It is evident that the problem differs in city and country communities, in manufacturing and agricultural communities, etc.; so that there will have to be several solutions, just as there must be several manuals to cover the flora of the United States.

3. Biological Grouping.

The proposition to submerge botany in a combination course of general biology is serious enough to be considered a problem. Historically it is a reversion to an abandoned method. This is no argument against it, except that it was tried once and found to be ineffective. One form of this demand is no problem at all, for the biological subjects are kept distinct and are simply bound together under one cover. This of course is not general biology in any real sense. The other form of the demand, which constitutes the real problem, is to ignore the distinctions of the various biological subjects, and to develop a knowledge of general biological principles by using plants and animals indiscriminately. Theoretically this has much in its favor, but practically it has never worked with immature students. For them the perspective is far too large to be grasped by their limited experience. This proposition is as far removed as possible from the demand for applied science, for it is pure biological science in its largest aspects. It is not necessary to present the arguments for and against it as an appropriate

method for secondary schools, for that is not my function here; and, besides, they will all be found in the educational literature of two decades ago.

4. The Point of Interest.

As a rule, the student in the secondary school, especially the boy student, can be interested in what he has learned to regard as valuable. He has little patience with studies whose value in his future activities he does not perceive. What is it good for, is the question most often in his mind in reference to his various studies; and botany seems to be one of the subjects that has not convinced him of its value. One of the important problems to-day is to discover the effective point of contact for botany with such students, so that the subject will obtain the consent of their minds. In many cases this contact may be on the economic side, but this appeal is too narrow, for it does not affect all communities, and probably would not touch the majority of secondary school students, massed as they are in the cities. The effective appeal must be general enough to fit into our civilization everywhere. I doubt whether anyone is in a position just now to suggest definitely the nature of this appeal; but to me just now it seems necessarily associated with plants as illustrating the fundamentals of physiology and with vegetation as a world feature. There should be formulated a clear statement of the real value of botany in secondary education, a statement from the point of view of the pupil rather than of the teacher. It should be a serious attempt to answer the boy's honest question, "What is it good for?"

Such are the problems to be attacked; and if such a thing is allowable at this stage, I would suggest that the preparation of teachers of botany be investigated, and the relation of that preparation to the standing of the subject in the various schools. This will help to settle what is now an impression, and it is the natural approach to the most important of our problems. If the relation between inadequate preparation and poor standing of the subject is clear, then the problem that confronts us is how to restrict the teaching to prepared teachers and how to provide a sufficient number of such teachers.

The problems as to the economic aspects of botany and the general biology proposition should be settled in conference, after a hearing of the various claims, and also after the collection of testimony as to the results of the application of the various schemes.

# THE PSYCHOLOGY OF INSECTS.

By Nathan A. Harvey,

*Normal College, Ypsilanti, Mich.*

I wish to state my reasons for believing that the mental processes of insects are qualitatively different from those of man and of other mammals. I mean by qualitatively different that they differ in kind. It is not difficult for us to understand that the mental processes of mammals, such as a dog or a horse, are of the same kind as those of man, although they differ quantitatively. If the mental processes of a dog or a horse were to be increased a very great deal, we might have such mental processes as man himself experiences. But with insects, such increase in quantity would not result in intelligence that would correspond to human intelligence, but would show a totally different kind.

I scarcely know how to illustrate what I mean by qualitative difference in mental processes, but perhaps it will make my meaning clear if I refer to the fact that we think under certain laws that we call laws of thought. These laws are the propositions that we call self-evident, or intuitive, such as the axioms of mathematics and such fundamental propositions as that a thing cannot be and not be at the same time. These are the laws of thought, without which it is inconceivable to us that any thinking can occur. If we can suppose a being not to be limited by these particular laws, but to think under a totally different set of limitations, we may be able to conceive what I mean by an intelligence that is qualitatively different.

The natural way of arguing this proposition would be to show first, that insects do manifest intelligence; and then to show that such intelligence is qualitatively different. But it seems in the present instance preferable to show that if insects do manifest an intelligence, it must be qualitatively different, and then to show that a high degree of intelligence, comparable in quantity to that of mammals, or even of man, is manifested.

The mental processes of man are conditioned by several factors. In the first place, they are conditioned by the brain and nervous system. For every mental process there is a corresponding physiological change, which change always, or nearly always, takes the form of the transmission of a nervous impulse through a nervous arc. We do not know anything about the nature of the connection between the nervous impulse and the

mental process which accompanies it, but it is necessary to suppose that the relation is a very important one; so important that if the nervous arc were different, the mental process would be different. Hence the transmission of a nervous impulse through a nervous arc that is different in structure and kind from that of human beings must of necessity be a mental process that is different in quality. The nervous system of an insect is not like the nervous system of a vertebrate. It is composed of a double nerve cord and ganglia ventrally placed, and neither by its structure nor by its position, which are the two tests that must always be applied, can it be shown to be homologous to the brain and spinal cord of man.

The simplest mental processes of man, which by their combinations and modifications make up all other mental processes, are sensations, whose concomitants are the transmissions of nervous impulses which are established in sense organs through brain centers. If our sense organs were different, there can be no doubt that our sensations would be different, and the resulting complex mental processes which have sensations for their elementary constituents would necessarily be different. We can show that the sense organs of insects are different from our own, and therefore we are justified in reasoning to the conclusion that the mental processes of which sensations are the elementary constituents must be different.

The eyes of insects are of two kinds. Each insect has two immovable compound eyes, which are composed of some thousands of facets, each of which is homologous to a single eye. If all the facets except one were to be covered up, the insect could still see out of that one. That this is the true interpretation is shown by the manner in which the compound eyes develop, as can be observed in successive changes of a growing caterpillar, and by the presence of hairs between the facets in many butterflies. The eyes are not movable as are our eyes, but the circle of vision is enlarged by the fact that each eye is convex, facets in different portions of the eye being directed toward different points of the compass. The structure of the eyes is not at all like our own. There are no liquids in the compound eyes, no crystalline lens, in fact, the structure is as different as can well be in organs that perform the same functions. It is probable that the compound eyes do not see, in the same sense of the word that our eyes see. They do not

define objects. By means of the compound eyes the insect can probably discern motion and color, but not define objects at all.

Besides the compound eyes, insects have three simple eyes, called ocelli. It is probable that these simple eyes can define objects, but at very small distances. They probably correspond to the kind of vision that we might have if we had a half inch simple lens placed in front of each of our eyes. It is evident, then, that the sensations of sight in insects must be different from our own sensations of sight.

Hearing is our next most important sense. It is doubtful if the most intelligent insects can hear at all. It is impossible to demonstrate the presence of an ear in the ants, bees, and wasps. In the grasshopper, the ear is found on the first segment of the abdomen, and in the cricket it is found on the tibia of the foreleg. Even where an ear is found, it is not at all like the vertebrate ear in its construction. The vertebrate, human, ear is a hydrostatic organ, the nerve terminations being distributed in a liquid. It may be questioned whether an ear that is not constructed with its nerve endings terminating in a liquid can properly be called an ear. At least we can say that, tested by position and structure, the ear in any insect where it is found is not homologous to the human ear. Hence we are privileged to infer that the sensations of hearing in insects are not the same sensations as are the sensations of hearing in man.

Taste and smell probably exist in insects, but it is doubtful if they can be considered as identical with the senses called by the same names in human beings. Smell in human beings is a very subordinate sense. It has its end organ located in the air passages, and has its nerve endings terminating in a mucous membrane. No sense of smell can be demonstrated in the openings of the air passages of insects, and that which is usually considered as the organ of smell consists of some hairs, usually on the antennæ. Neither by position nor structure can we consider the smell organs in insects homologous with the smell organs in man. It would seem that there must be an equally great difference in the sensations that are associated with the smell organs. In fact, it is doubtful if we can properly call such a sense the sense of smell, although it probably is true that the same characteristic of a body is perceived by the two senses, both called smell, one in the human being, and one in the insects.

The sense of feeling is located in the skin of the human being, and is the general sense out of which the other senses or sense organs seem to have been developed. The insect has its body coated on the outside with a hard skeleton, and the sense of touch must be experienced through the medium of touch hairs. Here the construction of the organs is of such a different character that it is with doubtful propriety that we call them by the same name. In fact, we may say that the sensations arising in the two kinds of organisms as the result of the activity of these two kinds of sense organs are qualitatively different.

We have now discussed the principal senses of the human body and in every case have reached the conclusion that similar senses are either absolutely wanting from insects, or appear in such a modified form that they are scarcely comparable to those of man. But insects do possess senses of a high degree of acuteness, and such senses are different from any that we possess. Experiments have shown that a cecropia moth can perceive the presence of a female at the distance of a mile or a mile and a half, under conditions where a man would be unable to perceive its presence at a distance of more than six inches. Shall we call this sense a new sense, or a heightened appreciation of the old? Such senses as are exemplified by the cecropia moth, and others that are shown to exist in bees, ants, and wasps seem to be located in the antennæ. But the antennæ are organs of which no homologue exists in the human body. Hence we are the more completely justified in asserting that the antennal senses, how many soever they may be, are such as produce no sensations in the human being.

The general structure of the body of the insect is different from that of man. Oxygen is carried to the tissues, not through the medium of the blood, but directly through air passages. Blood and breathing accompany many sensations in the human body, and a different set of sensations must accompany the different processes found in the body of the insect. The life is shorter, the manner of living different, the methods of maintaining existence are widely separated. If we recognize that a large part of our feelings, and consequently motives to action, are determined in us by the necessity of preserving ourselves from destruction in various contingencies, we shall be able to appreciate how very different must be the mental life of an insect.

It seems as if we had shown conclusively that if insects manifest intelligence it must of necessity be a very different kind of intelligence from that which man possesses. In the sense organs, and in the central nervous system, we find structures so exceedingly different that the resulting sensations and mental processes derived from them must be qualitatively different.

Insects do manifest a high degree of intelligence. There is no necessity for considering any insects except the bees, ants, and wasps, for among the two hundred thousand kinds already described, we shall find all degrees of intelligence, and none higher than that found in the bees, ants, and wasps.

The bees, ants, and social wasps have a highly differentiated social system. The community is differentiated into three different kinds of individuals, instead of merely two as in human society, and in some cases among ants we find four, five, or even six different kinds of individuals, which may be described as different sexes, or differentiated for the performing of functions as widely diverse as that indicated by the word sex. Such a differentiation of function and complexity of organization is the most emphatic indication of a high degree of life. When the differentiation is a social one, it seems to carry with it a high degree of intelligence.

All observers of ants, bees, and wasps concur in assigning to them a high degree of intelligence. If we were to see a company of horses or dogs performing the same kind of communal activities that bees do, preparing food of one kind for themselves and another kind for their young, caring for their young as bees do, producing wax and constructing from such material their comb, we should attribute to them without hesitation a large amount of intelligence. So when we observe a mud wasp building its cell, provisioning it with spiders that are not dead, but paralyzed by stinging in the ventral ganglia, when we see such complicated activities of this kind adapted so perfectly to a determinate end, we cannot escape the conviction that a high degree of intelligence is necessarily attributable to an insect that is capable of performing them. Whoever carefully studies any single individual of this group in all its varied activities throughout a period of days or weeks, cannot fail to come to such a conclusion. The only escape from it is to start with an undemonstrable supposition that these activities are the result not of intelligence but of instinct. By instinct is usually meant

a series of activities comparable to a reflex, performed without a knowledge of the end that is to come from the activities. It is usually employed to indicate a lack of intelligence, and the argument by which it is supported always involves a *petitio principii*.

The argument that the activities of insects are the result of instinct and indicative of a lack of intelligence, seems to arise from two preconceived notions: First, that no other animal than man is capable of manifesting intelligence. Second, that the activities are so complex it is inconceivable how they should have been learned and the results foreseen by any means that the insect possesses. How can the wasp, that has never seen a mud nest built, nor observed a parent wasp sting a spider, know how a spider should be stung, a mud nest built, or how to provide for the continuation of the species?

Everyone will acknowledge that the organization of an instinct depends upon the organization of the nervous system. Instincts are without any question well organized in insects. Insects have existed as a class from the time of the carboniferous age, and since the life of an average insect is not greater than a year, there have been many generations since that time. Thirty times as many generations of insects may occur in insects as can occur in man in the same number of years, and it is probable that insects have existed upon earth as a class thirty times as long as has man. Hence there has been time and abundant opportunity for such a variation of the nervous system in insects as would enable them to attain the degree of organization necessary for the fixation of instincts.

Man exhibits many instincts, but none of them are comparable in complexity nor fixedness to those of insects. But we do know that the more nearly perfect any activity becomes in us, the more nearly it approximates the character of an instinctive action. All of our activities that originate as conscious, voluntary, willed acts, by practice come to assume the form closely allied to that of instincts. We say that we instinctively raise our hand to ward off a blow. We may be unconscious of the process by which the act is started, but we are never unconscious of the result that is to come from the action. What we do perceive is the result of the action before the action occurs, and this is the motive to the act.

We may illustrate the action of a mental instinct by a refer-

ence again to the necessary, self-evident truths. These mental processes most nearly approximate an instinct. Were we asked how we know that the whole is equal to the sum of all of its parts, we immediately reply, "How else could it be?" So it is easy for us to conceive that if a mud wasp were to reply to the question how she knows just where to sting the spider and why she does that before putting it into the cell, she would say, "How else could it be done?" The point is that our knowledge of the fact that the whole is equal to the sum of all of its parts, which we know as soon as it is necessary for us to know it, is not an indication of a lack of intelligence, but an evidence of intelligence. A person who did not know it instinctively would be the one lacking in intelligence. So the activities that are called instinctive in insects are indications of a high degree of intelligence, and not of a lack of it.

There are some experiments that lead to the conclusion that insects are by nature stupid. Such experiments consist in confronting the insect with a problem which it is easy for a human intelligence to solve, and then when the insect fails to solve it, to declare that the insect is stupid. A classical example is that of Lubbock, who placed an ant in such a position that it could reach a much desired object by dropping half an inch, or in another case by piling up under the object a heap of dirt or other material. In both these cases the ant either failed to get the desired object or traveled a long way around to reach it. Hence the ant is declared stupid.

Let us imagine what would happen if a man were placed in such a situation, or confronted with a problem which it would be easy for an ant or a bee, with his special senses, to solve, or to obtain the information necessary to solve it, and the man, not being possessed of these senses or means of obtaining the data for solving the problem, should fail to solve it. The ant would not be justified in declaring the man stupid, or unintelligent.

The only interpretation for the diverse results, one obtained by observing ants and other insects in their occupations, and the other by putting them under experimental conditions, is to recognize that the mental processes of insects are qualitatively different from those of man. With this conclusion, we shall be much better prepared to study the activities of insects and to interpret their actions. It seems to me that a failure to recognize

this qualitative difference between the mentality of insects and that of man vitiates in a very large degree the work of the best comparative psychologists. Even Lloyd Morgan lumps together in one class the mental processes of insects and of man, failing to draw the distinction that I have pointed out here. Unless this distinction is drawn, we shall not be able to make progress in the study of insect mentality.

The above consideration will help us also in the study of the mental life of other animals. Recognizing that there is a qualitative differentiation to be observed in mental processes, we shall not try to interpret mental processes as being of the same kind when the nervous system is extraordinarily different. Especially, we shall not make the egregious blunder of assuming that we find in protozoa and in plants different degrees of mental processes that are qualitatively similar to those of man. This is what Binet has done in his "Psychic Life of Micro-organisms," and many other books are tinctured with the same dye. But the matter of psychic life in plants is another story.

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**Upon the initiative of the director of the mining school of St. Petersburg the council of professors has decided to start as a new section, geological prospecting, for preparing engineers who are specialists in prospecting various ores and examing crystals. Professors Fedorov, Bogdanovich, Nikitin, Yakovlev, and others will be at the head of this department. The necessity of such experts in Russia has long been felt. The meeting of geologists which took place in 1903 stipulated for the creation of such a line of study, and this decision seems to be near realization.**

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**"Nuts and their uses as Foods" is the title of U. S. Farmers Bulletin Number 332. It presents much most interesting and valuable information concerning the kinds and sources of nuts found in our markets, the average composition of nuts and nut products, digestibility of nuts, use of nuts, in foods of various sorts, and handling and marketing nuts. As an illustration of the data presented we may take the comparative statement of the composition of an oily nut (walnut) and a starchy nut (chestnut).**

| Oily nut (walnut). | Starchy nut (chestnut). |
|---|---|
| Water 2.5% | 5.9% |
| Protein 16.6% | 10.7% |
| Fat 63.4% | 7.0% |
| Starch, Sugar, etc. 13.5% | 71.5% |
| Crude fiber, 2.6% | 2.7% |
| Ash 1.4% | 2.2% |

O. W. C.

# THE DEVELOPMENT OF ALGEBRAIC SYMBOLISM FROM PACIUOLO TO NEWTON.

By Suzan R. Benedict,

*Teachers College, Columbia University.*

Introductory Note: In considering the improvement of our present algebraic symbolism, the possible elimination of certain signs that may have outlived their usefulness, the conservation of the best that have come down to us, the variation observed in passing from the literature of one country to that of another, and the possible unifying of the best of our present symbols, it has been found helpful to consider the early history of the subject. In particular, a study of the formative period from about 1500 to 1650 has been found of greatest value and it is to this period that Miss Benedict has given some attention. Since the subject is both valuable and interesting, and since her information has been secured from original sources available to but few teachers, I have asked Miss Benedict to present in condensed form the historical facts which she has secured. This she has undertaken to do, omitting much information commonly found in the histories of mathematics, and all discussion of the significance of the investigation. The result is a contribution to the history of mathematical education of such interest and value that this department is glad to publish it for the benefit of the profession. David Eugene Smith.

The growth of the symbolism of elementary algebra from its first appearance in print at the end of the fifteenth century to the time of Newton when it was practically complete, roughly divides itself into two periods, each about a century in length. There were in the sixteenth century two general methods of writing algebraic symbols, the one in use in Italy and Spain, and the other in the northern countries. Of these two systems the Italian is the older and therefore should have first attention.

The earliest printed work on algebra is contained in a volume published in Venice in 1494. It was written by a Franciscan monk who, after a youth of travel and adventure, retired to a monastery and there made for himself an honorable name through his works. This man was Luca Paciuolo, commonly known from his birthplace as Fra Luca di Borgo San Sepulcro, and his great work was entitled, *Summa de Arithmetica, Geometria, Proportione et Proportionalita.* The book is merely a

compilation made by a man with the interests of a teacher; and what has been said of his methods, that he gave little that was new, is also true of his symbols, many of which are found in the fifteenth century manuscripts.[1] But just as his summary of methods is valuable, so his table of *Caratteri Alghibratici*, the earliest in which they are systematically explained, is of great historic interest. The first character used is a radical sign, simply an abbreviation of the word *radix* or *radice*, signifying root. The symbol is Ꝛ and it is usually followed by abbreviations of the words *prima, secunda,* etc.; the first two and the last of the thirty roots being written as follows: Ꝛ.$p^a$., Ꝛ.$2^a$., Ꝛ.$30^a$. Besides using these forms, the second, third, and fourth roots are frequently written Ꝛ., Ꝛ.*cu.*, and Ꝛ.Ꝛ. and there is still another symbol, Ꝛ.*v.*, which Paciuolo calls *Radice Universale* and uses to indicate the root of a polynomial.

Among Italian writers of this period it was very common to indicate the omission of one or more letters by a line drawn above. So we find in the *Summa* the words for *plus* and *minus*, in Italian *piú* and *meno.*, indicated by p̃ and and m̃. In common with many of the early algebraists Paciuolo calls the absolute number *numero* and symbolizes it .*n*°. The unknown quantity was *thing*, in the Italian *cosa*, and from this word were derived the names *Coss* and *Cossic Art*, which at the opening of the Renaissance were synonymous with algebra. The name for the square was *censo.* Cossali, in his *Origine dell' Algebra,*[2] says that the word is probably of Latin derivation and was adopted by Leonardo of Pisa, but that its original meaning is unknown. *Cubo*, for the cube, needs no explanation, and for *relato*, a term signifying a power whose exponent is not a multiple of two or three, there seems to be none. The symbols used are abbreviations of the names: *.co.* for the first power, *.ce.* for the second, *.cu.* for the third, and .p̊.r̊., 2.r̊. etc., for the *relati.* All other powers are built up by a multiplicative plan through a combination of *.ce.* and *.cu.* with each other and with the *relati*, but it must be noticed that two *relati* are never written together; for example, the twenty-fifth power is not .p̊.r̊.p̊.r̊. as we might expect, but .8.r̊.[3]

[1] Several of these manuscripts have been examined in the preparation of this report, but no reference is made to them for the reason that they antedate Paciuolo.

[2] Vol. I, p. 12.

[3] For Paciuolo's symbolism, see *Summa*, f. 67, v.

Another great name, in connection with the Italian symbolism, is that of Tartaglia, to whom we owe the first solution of the cubic equation. In his *General Trattato* (Venice, 1556) he uses most of Paciuolo's symbols, but, since the first power and the first root are the same, he discards *.co.* and uses R̨. *prima* in its place. In this work, then, we find the powers of the unknown written as follows: R̨. *prima, ce, cu, ce.ce, rel, ce.cu,* $\frac{9}{2}$ *rel., etc.*[4] The roots are indicated by R̨. followed by *ce* or *cu*, as the case may be, and the universal radical appears as R̨.V. For *plus* Tartaglia writes the word *piu*, or more frequently the letter p. the upper part of which is nearly surrounded by a curved line, and his *minus* may be *men, me,* or $\tilde{m}$. But the most interesting bit of symbolism in the *Trattato* is the trace of the parenthesis in the expression *22 men* (*22 men* R̨ 6.) which is given as the product of

$$\sqrt{22 + \sqrt{22 - \sqrt{6}}} \text{ and } \sqrt{22 - \sqrt{22 - \sqrt{6}}}.$$[5]

The arithmetic of Antich Rocha (Barcelona, 1565), in which we find one of the earliest chapters on algebra written in the Spanish language, also uses the Italian symbols, although they are modified even more than in the *Trattato*. The radical sign does not appear, but the symbols for the powers of the unknown quantities, including the zero power or the absolute number, are as follows: *N, Co, Ce, Cu, Cce, R. CcCu, RR, Ccce, Ccu, Rce.* etc.[6] For the plus sign Rocha uses the word *Mas* or its abbreviation *ma*, and for the minus sign *me*, an abbreviation of *ménos*.

These three writers are by no means the only ones to use the Italian symbols. Many others, among them Cardan,[7] use all or part of them, but the books which have been mentioned may well be taken as types of the others. This symbolism was, however, restricted almost wholly to the south, and in the meantime there was growing up in the north a system which became far more popular. This is described in Stifel's edition of Rudolff's *Coss* (Nürnberg, 1553) and in Robert Recorde's *Whetstone of Witte* (London, 1557).

In Stifel's work the absolute number is called *dragma*, indicated by a modified *d*, and the unknown is *radix* whose symbol has never been explained.[8] There are, however, in Paciuolo's

[4] *Trattato*, Vol. II, f. 39, r.
[5] *Trattato*, Vol. II, f. 169, r.
[6] *Arithmetica*, f. 253.
[7] *Practica Aritmetica*, Milan, 1537.
[8] For Stifel's symbolism, see the table at the end of this report.

*Summa* several cases in which the radix is written with a small $r^9$. Reasoning from this fact and remembering that radix often means the unknown quantity as well as the root, it seems very possible that manuscript writers might easily have changed that symbol to the one in question. *Census* of the Italians is *zensus* with the Germans and the symbol 3 is merely the initial letter. For the cube we find a sign which again is evidently developed from the Italian symbol, for in manuscripts the letters of *cu* are frequently joined at the top, from which the evolution of the German form is an easy matter. Instead of *relato*, the Germans used *sursolidus*, from whose symbol, together with those for *zensus* and *cubus*, all the other powers are formed by a multiplicative plan. The + and — are seen here as signs of operation, and indeed Stifel is generally considered to have been the first to use them as such. He is not entitled to this credit, however, for they appeared "in a work by Grammateus in 1514,[10] and again in 1537" in a remarkable little book published in Antwerp by one Gillis vander Hoecke.[11] Stifel's radical sign is nearly like our own, appearing as $\surd$ followed by the symbol to show the index, and, in the case of the universal radical, separated from the quantity to which it refers by a large dot. But these were not the earliest forms, as appears from Stifel's commentary on Rudolff, who he says used a sign with three angles for the cube root and two for the fourth.[12] Recorde's *Whetstone of Witte*, which was the first algebra printed in English, and Masterson's *Arithmeticke* which followed it,[13] represent the powers of the unknown by symbols which differ from Stifel's only in that they are printed with much bolder type, and that *sursolidus* has a slightly different form. Masterson's radicals are also the same as Stifel's, but Recorde follows the plan of Rudolff. There was, nevertheless, one important contribution made by Recorde, namely, the equality sign, which he says is "a paire of parallels or Gemowe lines of one length, thus ═ becaufe noe. 2. thynges can be moare equalle."[14]

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[9] *Summa*, f. 128, v.

[10] The page from Grammateus is reproduced in Professor Smith's *Rara Arithmetica*, Boston, 1908, p. 125.

[11] The page from vander Hoecke is also reproduced in the same work, p. 185. The student may thus consult the fac similes of the original sources.

[12] Stifel's Rudolff's *Coss*, f. 82, v.

[13] London, 1592-94.

[14] *Whetstone of Witte*, f. Ff, j, v.

The first French work on algebra, that of Jacques Peletier (Paris, 1560), combines the Italian and the German symbolism. Here we find the German symbols for the cube, the fifth power, and the radical sign, but for the first power R̃ is used, while the square is $q$, and the plus and minus signs appear $\tilde{p}$ as and $\tilde{m}$.

One of the most important writers to use the German symbols was the Jesuit Christopher Clavius, whom we ordinarily think of as belonging to the Italian school, since he lived and taught in Rome. He was a native of Bamberg, however, and his German origin is very evident in his *Algebra* (Rome, 1608). Clavius uses Stifel's system in its entirety except that $N$ stands for the absolute number, and the parenthesis is used to show the universal radical. Such an expression as $\sqrt{3}(15+\sqrt{3144})^{15}$ occurs very often, a form which was probably suggested to him by Bombelli's *Algebra*.

Although during the sixteenth century most of the writers were using one or the other of these systems, or a combination of both, there were a few by whom its limitations were felt, and whose efforts at improvement are noteworthy. The first of these is Cardan, a man whose powerful intellect was felt in every phase of life. In his earlier work he adopted the Italian symbols, but in his masterpiece, the *Ars Magna* (Nürnberg, 1545), they are must less used. R̃ appears as the sign for the root, and *plus* and *minus* are still indicated by *p* and *m* although without the line above, and always separated from the term to which they refer by a colon. *Primo relato* is also used for the fifth power, but the symbol $R^{\circ}p^{\circ}$ differs from the one in common use. Here the similarity to the old method disappears, for the other powers of the unknown are indicated by the Latin words *res* or *positiones*, *quadratus*, and *cubus* which Cardan writes in full or abbreviates to suit the context, using the proper terminations to designate the Latin case. Thus, for example, the first power is sometimes *pot*$^{is}$ and sometimes *reb*$^{us}$, and the square may be $\tilde{q}d^{i}$, $qd^{\circ}$, or any one of a variety of other forms. Cardan used a multiplicative plan of combination, and his symbols are even more unsatisfactory than those he discards; but nevertheless the change is an evidence of his dissatisfaction with those in existence.

In this connection, there are two men whose work is most

[15] *Algebra*, p. 90.

important, namely, Raphael Bombelli of Bologna, and the Hollander Simon Stevin, both of whom used a method approaching the exponential system. Bombelli's *Algebra* was published in two editions, in 1572 and in 1579, which is evidence of its worth and appreciation. Here the powers of the unknown are expressed by the figures 1, 2, 3, etc., written above small arcs[16], and it is evident that Bombelli had some idea of the significance of the exponent, for he explains at considerable length that the product of $3^{1}$ and $10^{1}$ is $30^{2}$ because "the whole sum of 1 and 1 is $2^{17}$." These symbols, when used in the text, follow the coefficients, but in adding or subtracting polynomials they are often written above, thus furnishing an early example of detached coefficients. Bombelli's radical sign is simply an *R*, followed by *q* or *c*, to show the index, but in connection with this sign there is one very interesting point to be noticed, namely, the use of the adjective *legato* and its initial letter *L*. The word was not original with him for we find it in Cardan's arithmetic[18] where it signifies that the radical sign is to apply only to the term immediately following; for example *L* R 3 *p*: R 8 would mean $\sqrt{3} + \sqrt{8}$. Bombelli adopts the name and the symbol, but in his work, *legato* is synonymous with *universale*. Here we find the *L* written after the *R* and the expression is completed by placing an inverted *L* at the end[19] as in *R.q L* 128. p. $8^{2}$ ⌋. The Italian word *legato* means bound, or fastened together, and it is quite possible that our signs of aggregation may have had their origin in the initial letter *L*. In that case the idea of the bracket would not be due to Girard (1629), as the historians assert, but to Bombelli in 1572.

Almost contemporary with Bombelli, there lived in Holland another mathematician, Simon Stevin, who was working along the same line. Stevin was one of the best thinkers of his time and is well known as being among the first to use the decimal fraction. His symbols for the powers of the unknown are the same that he uses to indicate units, tenths, hundredths, and so on, in decimal fractions, namely small figures placed in circles[20]. If Stevin finds that several unknowns are necessary, he forms them by prefixing *sec* or *ter* to the ap-

[16] For Bombelli's symbolism, see the table at the end of this report.
[17] *Algebra*, p. 205.
[18] *Arithmetic*, f. A. 7, v.
[19] Bombelli's *Algebra*, p. 356.
[20] For Stevin's symbolism, see the table at the end of this report.

propriate symbol and often combines these in the same term. In this combination we often find an M used to denote multiplication, and there is a corresponding use of D for division which is common but by no means universal with Stevin. The radical sign appears in the German form, $\surd$ representing the square root, and the fourth and eighth roots requiring only the addition of a second and third angle. The symbol for cube root is the sign $\surd$ followed by the 3 in a circle,[21] which is modified by another angle to form the ninth root. These signs when placed before a monomial affect all of its factors unless they are separated from one another by a double arc forming a symbol which resembles an X, in which case the radical affects only the part preceding the sign. To indicate a root of a polynomial, Stevin writes the word *bino* or *trino* after the radical sign and we will frequently find such expressions as $\surd bino\ 2 + \surd 3$ or $\surd trino\ \surd 3 + \surd 2 - \surd 5$ for $\sqrt{2+\sqrt{3}}$ and $\sqrt{\sqrt{3}+\sqrt{2}-\sqrt{5}}$.

None of the early attempts to form a symbolism was really successful, and the first noteworthy improvement was made toward the close of the sixteenth century. This was accomplished by François Vieta, the greatest French mathematician of his generation, in whose mind was conceived the idea of a general symbolism by which known as well as unknown numbers might be represented. He wrote at different times during the last quarter of the century and much of his work has been lost; but from what was recovered and edited by Franciscus Van Schooten in 1646, a fair idea of his symbolism can be formed. In the theoretical part of his work the unknowns are expressed by the capital vowels and the known quantities by the consonants. To raise these to any power, the words *quadratus, cubus,* etc., or their abbrevations, are written after, as *A quad* 4, or *A cub* in *E plano* 2, for $4x^2$ and $2x^3y$. One of the treatises saved by Van Schooten is called *De Aequationum recognitione et emendatione,* and was first published after Vieta's death by his friend Alexander Anderson. In this paper appears still another symbolism which may or may not have been due to Vieta.[22] Here a fair beginning is made in the theory of equations and the statements are made in a symbolism which is generally known as Vieta's. But below each theoretical statement

[21] *Œuvres*, Girard edition, p. 19.

[22] For Cantor's discussion of this point see *Vorlesungen uber Geschichte der Mathematik*, Vol. II, p. 582.

there appears in smaller type a numerical equation as an illustration, with a symbolism altogether different. The unknown number is represented by N, the square by Q, and the cube by C, and these are combined by an additive plan to form the higher powers. For example Vieta says, "*If Acubus—B—D—G in Aquad+B in D+B in G+Din Gin A* equals *B in D in G*, then *N* is either *B, D or G*," and below it, by way of explanation, "If $1C-6Q+11N$ equals 6, then *N* is either 1, 2, or 3."[23] Another noteworthy feature is the vinculum, which was first used as a sign of aggregation in this work, and the symbol $\overline{\overline{\quad}}$, by which Vieta indicates the absolute difference.

These symbols of Vieta were used for many years although they underwent several modifications, notably in the work of Harriot.[24] In this work the *A* was changed to *a* and the powers were formed by a repetition of the letter, as in *aaa* for our $x^3$. Here also the equality sign is more commonly used than in any earlier algebra, and the signs $<$ and $>$ first appear to express inequality.[25] Harriott also uses a double bar in writing complex fractions, as in the case of $\dfrac{\dfrac{aaa}{b}}{\overline{\overline{d}}} = \dfrac{aaa}{bd}$ a symbolism not accepted by his successors.

Harriot's symbolism was not a bad one and might have remained in use until the present time had there not appeared, in the person of Descartes, a man whose authority was universally felt. In writing *La Géométrie* (Paris, 1637), Descartes felt it better to discard most of the symbols then in use, and to adopt for knowns and unknowns, respectively, the first and last letters of the alphabet as we use them to-day. The exponents which in this work are always positive and integral, are represented by small figures placed above and to the right, but in case of the square the letter is always repeated instead of being affected by the exponent 2. In dealing with the equation, Descartes usually transposes all the terms to the left side, marking by an asterisk the places of any powers that may be wanting, and representing the equality by the sign $\propto$, as in $x^5 * * * * - a^4b \propto o$.

[23] *Opera Mathematica*, p. 158.

[24] *Artis analytica*, London, 1631.

[25] "Comparitionis figna in feqentibus furpanda
Æqualitatis = vt $a = b$. fignificet $a$ æqualem ipfi $b$
Maioritatis $>$ vt $a > b$. fignificet $a$ maiorem quam $b$
Minoritatis $<$ vt $a < b$. fignificet $a$ minorem quam $b$."—p. 6.

With Descartes, therefore, our modern elementary symbolism became fairly perfected, and it needed only the authority of a Newton to make it permanent: Newton's *Arithmetica Universalis,* containing his lectures on algebra at the University of Cambridge, appeared in London in 1707. In the first few pages of this work the symbolism is explained. The symbols were not original with Newton but were essentially those of Descartes, with a few due to Harriot, Girard, and Recorde. The fact that he judged them worthy, however, and that he made use of them in one of his best known works, was sufficient to give them such standing that they are still the recognized language of algebra.

It would be easy to select from the works consulted numerous other symbols of interest which have been suggested, and which have served their purpose either for the time, or even until our own day. In Harriot's work, for example, some useful symbols of aggregation are introduced, and in Recorde's the equations are particularly interesting, but the space allowed for this report permits of reference to only a few of the more important symbols not generally studied at first hand even by certain writers on the history of the subject. It is hoped that the information given, fragmentary as it is, may be helpful to teachers of mathematics.

In conclusion, there is appended a table showing some of the symbols as they appear in the original works. The copy of Vieta consulted is in the library of Columbia University, but for an opportunity to examine the writings of the other mathematicians I am indebted to Professor David Eugene Smith, through whose kindness the books in his own library as well as many of those belonging to George A. Plimpton, Esq., of New York, have been placed at my disposal.

| Present Symbols | $\sqrt{}$ | $\sqrt{}$ | $\sqrt{}$ | $\sqrt{}$ | $\sqrt{}$ | $x$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | + | = |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Paciuolo | R.p$^a$. | R.2$^a$. | R.3$^a$. | R.4$^a$. | R.V | .co. Rprima | .ce. | .cu. | .ce.ce. | .p.p. rel | .p. ma | .m. me |
| Tartaglia | R | R | R | R R | R V: | Co | Ce | Cu | Cce | R | ma | me |
| Rocha | v | v | Rcu | R R | v. | Co | ce | cu | ce.ce | R | p | m |
| Stifel | v | v | w | w | v. | Co | ce | cu | ce.ce | R | p | m |
| Recorde | v | v | vce | vvv | v. | Co | ce | cu | ce.ce | R | p | m |
| Masterson | R | R | | | | Co | ce | cu | ce.ce | R | p | m |
| Peletier | v3 | v3 | vce | v3 3 | v3. | Co | ce | cu | ce.ce | R | p | m |
| de la Roche | R | R | | | | Co | ce | cu | ce.ce | R | p | m |
| Clavius | v3 | v3 | vce | v3 3 | v( ) | Co | ce | cu | ce.ce | R | p | m |
| Halcke | R□ | R | R□ | R□a□ | | Co | ce | cu | ce.ce | R | p | m |
| Ghaligai | R | R | cuR | | R V: | Co | ce | cu | ce.ce | R | p | m |
| Cardan | R.q | R | Rc | RR.q | RL J | Co | ce | cu | ce.ce | R | p | m |
| Bombelli | R$^{2a}$ | R$^{2a}$ | R$^{3a}$ | R$^{4a}$ | | Co | ce | cu | ce.ce | R | p | m |
| Vander Hoecke | v | v | v@ | w | v$_{bino}$ | Co | ce | cu | ce.ce | R | p | m |
| Stevin | v | v | vC. | w | v— | Co | ce | cu | ce.ce | R | p | m |
| Veta | v | v | v3) | | | Co | ce | cu | ce.ce | R | p | m |
| Harriot | v | v | vC. | v | v | a | aa | aaa | aaaa | aaaaa | p | m |
| Descartes | v | v | v | v | v | x | xx | x$^3$ | x$^4$ | x$^5$ | + | − |
| Newton | v | v | v | v | v | x | xx | x$^3$ | x$^4$ | x$^5$ | + | − |

## III MATHEMATICS.*

By Wm. T. Campbell,

*Boston Latin School.*

I will ask your attention to a single problem of mathematics —the problem of the great middle class. The "submerged tenth" of the mathematical population was a problem discussed during many years, until it was solved by the elective system. The problem of the middle class remains, more puzzling than hitherto, from the very cause which has removed the other problem from discussion. By the middle class I mean a body of students who have mathematical ability though they are not specialists, who could pursue mathematical studies farther than they do with profit to themselves and no discredit to their teachers. This class needs more consideration. Our best mathematical scholars—the upper tenth—are well provided for, and that class is satisfied; the poorest scholars—our former submerged tenth—have been freed from mathematics by the elective system, so that that class also is satisfied. Now an educational community, like a community in the social world, fares ill when the system under which it lives secures the happiness of the two extremes, but neglects the middle class on whom the strength of the whole must finally rest.

The extension of the elective system into secondary schools is bringing about the same situation which has existed in colleges for a long time. Commonly the first use made by pupils of the liberty of choice is to stop farther study of subjects which they find hard. Thus, just as already the college man may obtain his degree having studied no mathematics beyond algebra and geometry, so there is a prospect that the high school boy may go out into the world having studied no mathematics beyond arithmetic.

Are we ready to yield, and say that mathematics is for lovers of mathematics who will be self-impelled to pursue it from taste and conscious ability, and that we need not concern ourselves about the rest? Or shall we bestir ourselves to meet the situation? The elective system is established. Mathematics does not generally appeal to the young: its difficulties are manifest;

*Read before the joint session of the American Federation of the Mathematical and Physical Sciences and Section L of the American Association for the Advancement of Science.

its benefits are not so evident. We cannot claim, as teachers of languages can, that the obstacles lie at the beginning, and that afterwards each step forward is easier than the last. The fundamental qualities which mathematics demands at the outset and cultivates to the end—seriousness of purpose, resolution in attacking difficulties, continuous labor—are so many reasons to the young for avoiding the subject. What inducements can we offer to those students of good general ability who are choosing the line of least resistance, and are wandering from the path of mathematics?

We can profit more than hitherto by the example of classical teachers, who found themselves confronted by a similar situation when the classics ceased to be a protected subject. They set to work to reduce the drudgery, to economize time, and above all to bring in the element of attractiveness. Some years ago the head of the Greek department in the school where I teach was distributing to a class a new book. "This," said he, turning to me, "is a pleasing object." The book was, indeed, attractive to the eye; so I examined it with interest to see if the contents were equally pleasing. Again I was surprised: how skillfully the author had treated his subject, to make it attractive to others than classical enthusiasts. Drudgery was there in word lists to be learned by heart, forms, and rules of syntax; but it was so carefully distributed, so interspersed with interesting matter for translation, that it almost ceased to be drudgery. I had not then seen a mathematical book written with equal sympathy for the learner. There are some now. The idea is working—working upward from arithmetics which for some time have been written fully in that spirit.

Still, our mathematical text-books usually are too voluminous. The authors have been afraid to omit: it is so easy for the critic to point out what is lacking. As a result, text-books become formidable in size; and for this the efforts to enliven them with interesting matter are of themselves by no means a cure. Take the case of the graph in algebra. When the graph first appeared, it was welcomed as an attractive innovation: it would illustrate algebra and clear up obscure points; some thought it might replace some of the former methods. But now we find it presented in books with a development out of all proportion to the use made of it, and replacing nothing. Then the problems drawn

from physics. The substitution of these for some of the old ones has been a gain; but there appears to be a limit to their usefulness in algebra as it is now studied. When the physical idea is simple, they succeed; but when the solution of a problem requires more attention to the laws of physics than to the laws of mathematics, attractiveness fades.

It is the disposition of mathematical writers to intensify, to carry a topic too far, which is so discouraging to the average student. Hundreds can learn to control a machine in its ordinary use where one can drive it to its limits. When the mathematical machine is forced to its capacity, then is seen the joy of the specialist, but the despair of other students.

Lastly, just as we can profit more by the example of classical teachers in making our mathematical instruction more interesting, with the difficulties more judiciously distributed can we not also profit by the example of teachers of physical science? The study of that science has seen three stages: the book stage, when students read about experiments and were aided only by diagrams: the illustrated stage, when the students looked on while the instructor performed the experiments; and the present practical stage, when the students perform the experiments themselves. Methods of mathematical instruction, improved on the lines now followed, will leave us at the best only in the second stage. Indications seem to point to the need of a more practical basis for both algebra and geometry than we are now attempting—perhaps to a kind of mathematical laboratory. The situation in algebra is much clearer than in geometry, and the way seems open to a closer connection with other sciences if these can begin earlier in the course, so that many problems which now are unsuitable may become truly real and practical. In geometry, the solution may lie in a combination of practical constructions, mechanical drawing, and practical applications of geometric principles—all of which are already in the common course though separately and in elementary forms.

If, however, the mathematical laboratory comes, let it come, not as an addition to what we are now doing, but as a substitute for parts of it—in the interest of the majority of those now being educated, a majority that no longer seeks education for mental discipline, for culture, for preparation to leadership, but as a means of gaining a better living.

## PROBLEM DEPARTMENT.

IRA M. DELONG,

*University of Colorado, Boulder, Colo.*

***Readers of the magazine are invited to send solutions of the problems in this department and also to propose problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to Ira M. DeLong, Boulder, Colo.***

### Algebra.

139. Find whether the following series is convergent or divergent.

$$\frac{1}{x}-\frac{1}{x+a}+\frac{1}{x+2a}-\frac{1}{x+3a}+\cdots\cdots$$

$x$ and $a$ being positive.

*I. Solution by Orville Price, Denver, Colo.*

The series is convergent (see Wentworth's College Algebra, § 374). It may be written as follows:

$$\frac{1}{x}-\left(\frac{1}{x+a}-\frac{1}{x+2a}\right)-\left(\frac{1}{x+3a}-\frac{1}{x+4a}\right)\cdots\cdots,$$

showing that the sum is always less than $\frac{1}{x}$.

*II. Solution by I. L. Winckler, Middlebury, Vt.*

Let the terms of the series be represented by $u_1, u_2, u_3 \ldots\ldots u_n$

Then the series may be written in the two forms,

$(u_1-u_2)+(u_3-u_4)+\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

and $u_1-(u_2-u_3)-(u_4-u_5)\ldots\ldots\ldots\ldots\ldots\ldots(2)$

Since $u_1>u_2>u_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

(1) shows that the sum of any number of terms is positive and greater than $u_1-u_2$. (2) shows that this sum is less than $u_1$.

Therefore the series is convergent.

140. *Proposed by W. T. Brewer, Quincy, Ill.*

Solve $x^3y+y^3x=a\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

$x^2-y^2=b\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$

*I. Solution by A. M. Harding, Fayetteville, Ark.*

Square (2) and we have $x^4-2x^2y^2+y^4=b^2$ or $x^4+6x^2y^2+y^4=b^2+8x^2y^2$ (3)

Multiply (1) by 4 and we have $4x^3y+4y^3x=4a\ldots\ldots\ldots\ldots\ldots\ldots(4)$

Add (3) and (4) $x^4+4x^3y+6x^2y^2+4xy^3+y^4=b^2+4a+8x^2y^2\ldots\ldots(5)$

Subtract (4) from (3) $x^4-4x^3y+6x^2y^2-4xy^3+y^4=b^2-4a+8x^2y^2\ldots(6)$

$$\left.\begin{array}{l}\therefore (x+y)^4=b^2+4a+8x^2y^2\\ (x-y)^4=b^2-4a+8x^2y^2\end{array}\right\}\text{Multiply}$$

$(x^2-y^2)^4=b^4+16b^2x^2y^2+64x^4y^4-16a^2$

or $b^4=b^4+16b^2x^2y^2+64x^4y^4-16a^2$

$\therefore 8x^2y^2=-b^2+\sqrt{b^4+16a^2}.$

Then $(x+y)^4=4a+\sqrt{b^4+16a^2}$

$(x-y)^4=-4a+\sqrt{b^4+16a^2}.$

$$\therefore x = \frac{(\sqrt{b^4 + 16a^2} + 4a)^{\frac{1}{2}} + (\sqrt{b^4 + 16a^2} - 4a)^{\frac{1}{2}}}{2}$$

$$y = \frac{(\sqrt{b^4 + 16a^2} + 4a)^{\frac{1}{2}} - (\sqrt{b^4 + 16a^2} - 4a)^{\frac{1}{2}}}{2}$$

*II. Solution by Walter L. Brown, Fancher, N. Y.*

Let $x=z+u$, $y=z-u$, $a=2c$, $b=4d$ then $z^4-u^4=c$, $zu=d$, $4z^4u^4=4d^4$
$z^8-2z^4u^8=c^2$, $z^8+2z^4u^4+u^8=c^2+4d^2$

$$z^4 + u^4 = \sqrt{c^2 + 4d^2},\ z,\ u = \left\{\tfrac{1}{2}\sqrt{c^2 + 4d^2} \pm \tfrac{1}{2}c\right\}^{\frac{1}{4}}$$

$$x = \left\{\frac{1}{2}\sqrt{\frac{a^2}{4} + \frac{b^4}{64}} + \frac{1}{4}a\right\}^{\frac{1}{4}} + \left\{\frac{1}{2}\sqrt{\frac{a^2}{4} + \frac{b^4}{64}} - \frac{a}{4}\right\}^{\frac{1}{4}}$$

$$y = \left\{\frac{1}{2}\sqrt{\frac{a^2}{4} + \frac{b^4}{64}} + \frac{1}{4}a\right\}^{\frac{1}{4}} - \left\{\frac{1}{2}\sqrt{\frac{a^2}{4} + \frac{b^2}{64}} - \frac{a}{4}\right\}^{\frac{1}{4}}$$

*III. Solution by W. T. Brewer, Quincy, Ill.*

Solve $x^2y+xy^2=a$ .....(1)

$x^2-y^2=b$ .....(2)

From (1) $x^2+y^2=\dfrac{a}{xy}$ .....(3)

Subtract sq. of (2) from sq. of (3)

$$4x^2y^2 = \frac{a^2}{x^2y^2} - b^2$$

$$\therefore xy = \tfrac{1}{4}\left(-2b^2 \pm 2\sqrt{16a^2 + b^4}\right)^{\frac{1}{2}}$$

Substituting this value in (3) and reducing

$x^2+y^2=\pm\ \frac{1}{2}\left(2b^2 \pm 2\sqrt{16a^2 + b^4}\right)^{\frac{1}{2}}$ .....(4)

From (2) and (4) $x=\pm\ \frac{1}{2}\left(\pm\sqrt{2b^2 \pm 2\sqrt{16a^2 + b^4}} + 2b^2\right)^{\frac{1}{2}}$

$y=\pm\ \frac{1}{2}\left(\pm\sqrt{2b^2 \pm 2\sqrt{16a^2 + b^4}} - 2b^2\right)^{\frac{1}{2}}$

## Geometry.

141. *Proposed by A. Latham Baker, Ph.D., Brooklyn, N. Y.*

Given P, Q, R, the three angles of a triangle, with the mid-point of one side at a fixed mid-point between two parallels, and the two vertices on the parallels. What is the locus of the third vertex?

*Solution by E. E. Wood, Fort William, Ontario.*

Through M, the mid-point between the parallels, draw $A_1B_1$ perpendicular to the parallels, meeting one of them in $A_1$ the other in $B_1$. On $A_1B_1$ as a base construct the triangle $A_1B_1C_1$ with angles equal respectively to P, Q, R.

Draw $A_2B_2$ through M, meeting the same parallels in $A_2$, $B_2$. On $A_2B_2$ construct the triangle $A_2B_2C_2$ with angles equal respectively to P, Q, R.

Join $MC_1$ and $MC_2$, and let angle $B_2MB_1=\theta$; then $C_2MC_1=\theta$

Now $\dfrac{MB_1}{MC_1} = \dfrac{MB_2}{MC_2}$, or $\dfrac{MB_1}{MB_2} = \dfrac{MC_1}{MC_2}$, and since each pair contains

the angle $\theta$, the triangles $MB_1B_2$, $MC_1C_2$ are similar. Therefore $MC_1C_2$ is a right angle, and the required locus is the line $C_1C_2$ perpendicular to $MC_1$ at the point $C_1$.

142. *Proposed by D. W. Duguid, Dansville, Mich.*

To construct the expression $x = \frac{2abc}{de}$; that is $\frac{2ab}{d} \times \frac{c}{e}$.

(Wentworth's Geometry, revised edition.)

*Solution by Wm. B. Borgers, A.B., Grand Rapids, Mich.*

Draw two indefinite lines including any convenient acute angle. On one side lay off $OA=d$, $AB=2a$. On the other, $OC=b$. Draw AC. Then draw BD parallel to AC. Then $CD=2ab/d$. Through C draw CE making any convenient angle. Take $CE=e$, $EF=c$. Draw ED, and then draw FG parallel to ED. Then $DG=x$.

## Applied Mathematics.

143. *Proposed by O. R. Sheldon, Chicago, Ill.*

A ball is placed upon a given horizontal plane. Determine its position in the time $t$.

*Solution by Philip Fitch, Denver, Colo.*

Let C be the center of the earth, MN a plane tangent to the earth at T, and P the point where the ball is initially placed on the plane. Draw AB perpendicular to MN at P and join TC; the lines AB and TC are parallel. Pass a plane through TC and AB; it will contain the lines TP and CP. Since the force along CP can be resolved into two components, one perpendicular and the other along the plane MN, the ball will roll along the intersection of the plane MN and the plane C—AB (assuming it to be at rest at P). Let S be any position of the ball while moving along TP; let $SC=d$, angle $TCS=\theta$, angle $TCP=\rho$, $TP=a$, $TS=x$. The force of gravity acting is $G=-\frac{k}{d^2}$. The component $f$ along PT is $f=-\frac{k}{d^2}\cdot\frac{x}{d}=\frac{-kx}{\left(x^2+R^2\right)^{\frac{3}{2}}}$.

Then $$\frac{d^2x}{dt^2}=-\frac{k}{m}\frac{x}{\left(x^2+R^2\right)^{\frac{3}{2}}} \qquad \ldots\ldots(1)$$

$$\frac{dv}{dt}=\frac{-1}{2}\frac{x}{\left(x^2+R^2\right)^{\frac{3}{2}}},\ \left(\frac{1}{2}=-\frac{k}{m}\right) \qquad \ldots\ldots(2)$$

From (2), $$2vdv=-1\ \frac{xdx}{\left(x^2+R^2\right)^{\frac{3}{2}}}$$

Integrating, $$v^2=\frac{1}{\sqrt{x^2+R^2}}+c.$$

Since $x=a$ when $v=0$,

$$c=-\frac{1}{\sqrt{a^2+R^2}}=J,$$

$$v=\pm\sqrt{\frac{1}{\sqrt{x^2+R^2}}-J} \qquad \ldots\ldots(3)$$

Since $v = \frac{dx}{dt}$, we find

$$t = \pm \int \frac{\left(x^2 + R^2\right)^{\frac{1}{4}} dx}{\left[1 - J\left(x^2 + R^2\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}} + C_3.$$

with the initial conditions $x = a$ when $t = 0$.

## CREDIT FOR SOLUTIONS RECEIVED.

Algebra 134. E. E. Wood. (1)

Algebra 135. John Gaub. (1)

Geometry 136. E. E. Wood. (1)

Algebra 139. T. M. Blakslee, Walter L. Brown, W. A. Challacombe, John Gaub, Louis Lindsey, W. L. Malone, Orville Price, I. L. Winckler, E. E. Wood, G. B. M. Zerr. (10)

Algebra 140. A. M. Allison, T. M. Blakslee, W. T. Brewer, Walter L. Brown, W. A. Challacombe, John Gaub, W. L. Malone, M. H. Pearson, Orville Price, O. R. Sheldon, I. L. Winckler, E. E. Wood, G. B. M. Zerr. (13)

Geometry 141. A. Latham Baker, T. M. Blakslee, Wm. B. Borgers, J. S. Brown, Walter L. Brown, Philip Fitch, W. L. Malone, Orville Price, I. L. Winckler, E. E. Wood, G. B. M. Zerr. (11)

Geometry 142. A. M. Allison, A. Latham Baker, Ira P. Baldwin, T. M. Blakslee, Wm. B. Borgers, J. S. Brown, Walter L. Brown, W. A. Challacombe, Byron Cosby, W. L. Malone, M. H. Pearson, James H. Weaver, I. L. Winckler, G. B. M. Zerr. Two unsigned solutions, one dated New Albany, Ind., Jan. 29, 1909; the other Denver, Colo., Feb. 8, 1909. (16)

Applied Mathematics 143. Walter L. Brown, Philip Fitch, O. R. Sheldon, G. B. M. Zerr. (4)

Total number of solutions, 57.

## PROBLEMS FOR SOLUTION.

### Algebra.

149. *Proposed by I. L. Winckler, Middlebury, Vt.*

$$x(y+z-x) = a$$
$$y(z+x-y) = b$$
$$z(x+y-z) = c$$

150. *Proposed by G. B. M. Zerr, Ph.D., Philadelphia, Pa.*

In a certain country the tax per $1 on a person's income varies as the fourth root of the number of dollars and when the income is $10,000 the rate per dollar is 10 cents. Find the largest net income possible.

### Geometry.

151. *Proposed by W. A. Challacombe, Carlinville, Ill.*

Given two circles, with centers O and O′, and a point A in their

plane, to draw through the point A a straight line, meeting the circumferences at B and C so that AB : AC = $m$ : $n$. (From Wentworth's Plane Geometry.)

152. *Proposed by Lloyd Holsinger, Peoria, Ill.*

In a circumscribed triangle the lines joining the vertices to the points of contact are concurrent.

### Applied Mathematics.

153. *Proposed by G. W. Greenwood, Dunbar, Pa.*

Find the total force against a dam 30 feet long when the water stands 20 feet higher on one side than on the other. (From Millikan & Gale's Physics.)

154. *Proposed by Ira P. Baldwin, Emporia, Kansas.*

On a level plain the crack of a rifle and the thud of the ball striking the target are heard at the same instant. Find the locus of the hearer. (S. L. Loney's Coördinate Geom., p. 283.)

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## PERSONALS.

George Sype, instructor in Township High School at Pontiac, Ill., for the past three years, has recently gone to the mathematics department of the Austin High School, Chicago, Ill.

Mr. V. D. Hawkins, for several years one of the instructors in physics and mathematics in the Joliet (Ill.) High School, has accepted a position as instructor in mathematics in the new Technical High School, Cleveland, O.

Mr. Geo. A. Abbott, who has been head of the department of chemistry in the Manual Training High School, Indianapolis, Ind., has been appointed assistant professor of organic and industrial chemistry in the North Dakota Agricultural College at Fargo.

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A section of the American Chemical Society was formed in Cleveland, January 11, 1909. Franklin T. Jones of University School was elected chairman. The chairmen of local sections are vice-presidents of the American Chemical Society.

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## OBITUARY.

Professor John Joseph Browne, a valued contributor of the Problem Department, died in Denver, Colo., February 20, 1909. He was born in Ireland 42 years ago. He graduated from the Royal University of Dublin, having enjoyed there the rare privilege of taking lectures under Dr. George Salmon. Coming to Colorado for his health in 1901, he was for two years Assistant in Mathematics in the University of Colorado, entering then the Faculty of the Colorado School of Mines, first as Instructor in Mathematics, and later as Assistant Professor of Mathematics.

## ANGLES.

G. W. GREENWOOD,

*Dunbar, Pa.*

It is curious that a definition proposed in the October number of SCHOOL SCIENCE AND MATHEMATICS (page 576, last paragraph) for "the angle between two straight lines" is in the January number (page 45) confused with, and defended as, a definition of an angle! Now the idea of an angle must precede the ideas of angles associated with straight lines, curves, planes, etc., just as the definition of a circle inscribed in a triangle presupposes the idea of a circle. It appears to the writer that an angle is, fundamentally, a figure formed by two non-collinear rays having a common origin, and that no conception of an angle can be simpler. Such a definition, recommended by the Committee on Geometry, is used by Hilbert in his Foundations and Halsted in his Rational Geometry. "It is questionable whether the measure of such a figure, without further explanation, would convey any clear notion to the student." The same is true of the measure of the surface of a sphere, not to mention the surfaces familiar solids where measurement is impossible. "It is evident that it would be desirable that a definition of an angle should be so formulated that it would appear at once natural to speak of the measure of an angle in terms of some unit of measure." In the case of a sect this attempt would be hopeless, since some sect is necessary before measurement is possible; the same is quite probably true of an angle, as a glance at a clock will convince one that it is the *angle* that measures the *rotation*.

In considering "*the* angle *between* two straight lines" we note that the idea of "betweenness" in connection with non-parallel lines is rather vague; where should a point be located in order to lie "between" two intersecting lines? In the case of intersecting lines, *the* angle is rather ambiguous; if the lines are parallel there is no apparent angle, and, to use the words of the proposer of this definition, "it is questionable whether" such a definition, "without further explanation, would convey any clear notion to the student."

In case it is desired to establish a relation between two straight lines in terms of an angle, the writer would suggest the following, *after* the consideration of parallels: Consider any two undirected straight lines, $a$ and $b$. From any point O of $a$ draw a ray $x$ parallel to $b$. If $x$ is perpendicular to $a$, $a$ and $b$ are said to be perpendicular (whether or not they are coplanar). If $x$ is oblique to $a$, the *acute* angle determined by them is called the *inclination* of $a$ to $b$, or $b$ to $a$, the measure of the inclination being the measure of the acute angle. If $x$ coincides with a ray of $a$ from O, $a$ and $b$ are parallel and we *say* that the inclination of each to the other is zero. This last statement extends by a postulate or definition the idea of inclination to parallels.

The modifications of this method in the case of directed lines are obvious. In the general case of undirected sects the attempt to define the corresponding angle free from ambiguity would doubtless prove futile.

*Dunbar, Pa.*

## SCIENCE QUESTIONS.

FRANKLIN T. JONES,

*University School, Cleveland, Ohio.*

*Please send lists of the laboratory exercises in physics you will use this year. Do you have your students calculate per cents of error? If not, how long ago did you abandon it?*

6. *Proposed by A. Haven Smith, Sioux City, Iowa.*

What would be the effect of Doppler's principle applied to heat? to light?

7. *Proposed by O. R. Sheldon, Chicago, Ill.*

When a ship is struck by a ball fired at very great velocity, splinters are not scattered. Why?

8. *Proposed by the Editor.*

Is the present departmental system in large high schools not likely to produce narrow kind of specialization on the part of the teacher? [Frequently one teacher instructs as many as six classes in succession on the same lesson. Compare this specialization with that of the college instructor. Which is the more narrowing?]

*J. Hawley Aiken, Saratoga Springs, N. Y.*

A solution of the problem regarding the coiled steel spring dissolved in acid.

We say that the coiled spring has potential energy and that this energy is changed to kinetic energy as the spring is released.

If the spring is coiled the molecules are under stress, and as the spring is dissolved the particles pass out into the solution with greater than the usual velocity, thus producing an amount of kinetic energy in the form of molecular motion, equal to that which would have been produced by the uncoiling of the spring.

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## EXAMINATION PAPERS FROM VARIOUS SCHOOLS.

### Semester Examination in Physics, Englewood (Chicago) High School January, 1908.

1. Define energy, weight, efficiency, temperature, vector.

2. A body in air weighs 150 g., in water 110 g., in alcohol 100 g. Find volume of body, specific gravity of body and of the alcohol. State principles.

3. State Newton's laws of motion; give an application of each. How far will a body fall in 9 seconds?

4. Change 60° C. to F°. Change 98° F. to C°. What is a calorie? Absolute zero? Temperature? Heat?

5. How are some bodies made stable? Describe a barometer and tell how and what it may tell. What is "normal pressure?"

6. Draw a diagram of and explain the action of a lift pump. Find the resultant of two motions of 40 ft. east and 20 ft. south.

## Examination Questions in Chemistry Given at End of First Term, January, 1909, Central High School, Kansas City.

FREDUS N. PETERS.

1. Define the term catalysis, and give two instances to illustrate.

2. Give two methods used in preparing oxygen and write equations illustrating.

3. Complete the following equations and state what experimental work they represent:

$$NH_4Cl + Ca(HO)_2 \rightarrow$$
$$NH_4NO_3 + NaHO \rightarrow$$
$$NaCl + H_2SO_4 \rightarrow$$

4. How much sulphuric acid would be needed to prepare 2400 lbs. blue vitriol?

(*b*) 10,000 cc. of gas collected over water in a room at 20 degrees C. and barometer reading 740, would have what volume under standard conditions?

5. Name the two substances generally used in the laboratory in making chlorine. Write the equation. Name three other substances any one of which might be substituted for one of the above. Explain why any one of these may be substituted.

6. Explain how chlorine bleaches: write the equation to illustrate. Why does printer's ink not bleach? Copper sulphate?

7. Copy and give names: $H_3PO_4$, $H_2BaO_2$, $H_2CO_3$, $Na_3PO_4$, $Na_2HPO_4$, $P(HO)_3$, $Cu_2O$, $CuO$, $FeCl_2$, $Fe_2Cl_6$.

8. Copy and write formulas: phosphorous acid, mono-sodium phosphate, potassium carbonate, hydrosulphuric acid, sodium sulphide.

9. Give the preliminary and final tests for a suspected bromide and write all equations illustrating.

10. Explain meaning of the terms, absolute zero, standard pressure.

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## Physics Examination, Heat (50 min.), Springfield (Mass.) Technical High School.

1. The temperature of this room is 18°C. What is it on the Fahrenheit scale?

2. The barometer now reads 76.6 cm. What is the boiling point of water?

3. Explain how a glacier moves down a mountain valley. What experiment was made in the lecture room to illustrate this principle?

4. Show by a diagram the convection currents that exist in a well-designed refrigerator.

5. What practical application of expansion, specific heat, and latent heat do you see in steam and hot water heating systems?

## DEPARTMENT OF METROLOGY.

### Concerning Coinage.

In France copper is about to give place to aluminum for minor coins. More than 500,000,000 of the five-centimes and nearly 350,000,000 of the 10-centimes denominations are to be coined from the white metal to take the place of similar copper coins. This is one of several uses in which the lighter metal can be substituted for the heavier. Other nations will watch the experiment with interest.

On the other hand we read that the British mint has lately been turning out copper coins at the rate of four tons a day to supply a famine in pence in London and other large centers.

The mints of the U. S. coined, during 1908, almost $200,000,000 worth of gold alone. Some of this of course was recoinage of abraded coins, as always happens. In 1888, 20 years earlier, the gold coinage was only $31,000,000. What have such facts as these to do with teachers? Very much more than most people think. Let us take a few more facts. In 1906 the world's production of gold was more than $400,000,000. In 1886, twenty years before, it was little over a quarter as much, viz., $106,000,000. In 1886 the United States *produced* less than $35,000,000, in 1906 more than $94,000,000, an increase of over 170%. The enormous recent increase in the gold production in South Africa, Alaska, Colorado, and Nevada (which state last year produced about $17,000,000, and is said to have the largest gold mine in existence) has very much to do with increased cost of living.

Experts tell us the increase in cost of the commodities of life in the last five years has been 40%. Everybody knows it has been great. Prediction has also been made that within the next five years a further increase of 33% would follow. An increase of over 70% in living expenses within 10 years is enough to make a small-salaried teacher think a second time. Gold is the world's standard for measuring values. It is gold in one scale-pan balancing all other commodities in the other pan, and in that pan must be put not alone that used for coinage, but the amount for all other uses, jewelry, plating, watches, teeth-filling, etc.

With such increases in the precious metal it is impossible to keep the commodity pan from rising, in other words, prices from going up. Higher and higher will go the cost of living as more and more of the yellow metal is dug from the earth. It is not pretended this is the whole story. It is one large factor. Thirteen years ago, when free silver was agitated, economists said a tremendous inflation would follow a "silver standard," and then a terrific collapse—which was undoubtedly true. Now we are having a great and protracted gold inflation. Will the deluge follow? R. P. W.

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**The mine operators and miners' associations in England are advocating the establishment of a Mines Dept. with a Minister of Mines in the Cabinet.**

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**Concrete posts and cross-supports are being used in mine work with marked advantage.**

## REMARKS ON CERTAIN FALLACIES.

Similar to the common proof that 2=1 is the following proof that 3=1; we may build up others, as interesting exercises in Algebra:

$$\text{Let } x=a \quad (1)$$
$$\text{then } x^2=a^2 \quad (2)$$
$$\text{and } x^3=a^2x \quad (3)$$
$$\text{also } x^3-a^3=a^2x-a^3 \quad (4)$$
$$=a^2(x-a)$$
$$\therefore x^2+xa+a^2=a^2 \quad (5)$$
$$\text{substituting } a \text{ for } x,$$
$$3a^2=a^2 \quad (6)$$
$$\text{or } 3=1 \quad (7)$$

It may well be said that the fallacy consists in assuming that (5) is true; while in fact we have divided equals by $x-a$, or 0, which is meaningless. Is it not helpful to note that in (2) we have introduced the root $x=-a$, and in (3) the root $x=0$, then to get (5) from (4) we throw out the original root $x=a$, but to get (6) we use again $x=a$? If in (5) we had substituted either $x=-a$ or $x=0$, the only remaining values of $x$, we should have had $a^2=a^2$, or 1=1.

II.

A pail half full=a pail half empty (1)

$\therefore$ A pail full=a pail empty (2). Multiplying equals by 2.

The fallacy consists in assuming that equations compare materials, whereas they compare numbers. Of course the number of cubic inches in one half the volume of a pail equals the number of cubic inches in the other half. In this sense a pailful equals a pail empty.

III.

Some writers, in deriving the equation of a conic, define an ellipse as a curve such that the sum of the distances of any of its points P from two fixed points $F^1$ and F equals a constant, $2a$; thus $PF^1+PF=2a$ (1); for the hyperbola the difference, $PF^1-PF=2a$ (2); but in squaring, transposing rational terms, and squaring again they get from (1) the same equation as from (2), an equation equivalent to the four equations $\pm PF^1 \pm PF=\pm 2a$, but not equivalent to (1) or (2). So the difference between the two definitions is entirely lost in the non-reversible process of squaring. Have not the blind been leading the blind?

*Goshen College.* D. A. LEHMAN.

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## TEACHERS OF HIGH SCHOOL MATHEMATICS, PLEASE READ.

The Committee of the Central Association of Science and Mathematics Teachers on Unifying Secondary Mathematics, want the names of all teachers who are attempting to introduce a course in mathematics in which arithmetic, algebra and geometry are taught as much as possible as closely related parts of one subject, mathematics. They urge any teacher who is engaged in working out such a course, or who knows some one who is, to communicate the fact to the committee. Address Walter W. Hart, Shortridge H. S., Indianapolis, Ind.

## REAL APPLIED PROBLEMS IN ALGEBRA AND GEOMETRY.

*Committee on Investigation:* JAMES F. MILLIS, *Chairman, Francis W. Parker School, Chicago;* JOS. V. COLLINS, *State Normal School, Stevens Point, Wis.;* C. I. PALMER, *Armour Institute of Technology, Chicago;* E. FISKE ALLEN, *Teachers College, New York, N. Y.;* A. A. DODD, *Manual Training High School, Kansas City, Mo.*

### ANNOUNCEMENT.

This committee of the Mathematics Section of the Central Association of Science and Mathematics Teachers earnestly solicits the coöperative efforts of teachers of mathematics and others in the attempt to collect for use in secondary schools the *real applied problems* of elementary algebra and geometry in the various vocations, industries, sciences, etc., and to assist in any way in determining the extent to which such problems exist in the various fields. Teachers are urged to secure the aid in this investigation of men engaged in the various practical pursuits. By working together in this way, it should be possible in a short time to place in the hands of all teachers of secondary mathematics a valuable collection of real problems for class room use. For a fuller account of the work of this committee, see the March number of SCHOOL SCIENCE AND MATHEMATICS.

*All real applied problems of algebra or geometry that are sent to the members of the committee each month will be printed in* SCHOOL SCIENCE AND MATHEMATICS, *and proper credit given the contributor. Any one having only one or two problems even is urged to send them to the committee.*

### PROBLEMS.

*By J. F. Millis, Francis W. Parker School, Chicago.*

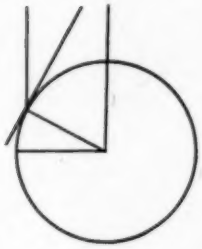
1. The potential energy (in foot-pounds) of a fly-wheel that drives a punch to punch holes in metal is computed by the formula

$$E = W\,\frac{V^2 - V_1^2}{64}$$

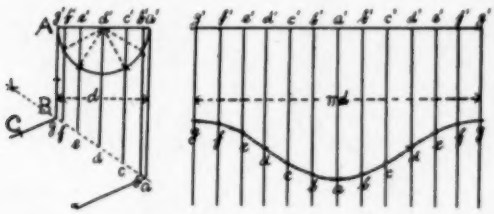
where E is potential energy, W is weight of rim of fly-wheel, V is velocity of center of gravity of fly-wheel (in feet per second) before impact of punch, and $V_1$ is velocity after impact. A fly-wheel makes 180 revolutions per minute, its center of gravity describes a circle 36 in. in diameter, the weight of rim is 1,200 lb., and the velocity is reduced 20% at each impact of the punch. Compute its potential energy.

2. The potential energy, E, required of a fly-wheel that operates a punch to punch a 1-in. hole through a steel plate ¾ in. thick is 2,950 foot-pounds. By the formula in Prob. 1, compute the weight of fly-wheel that is designed to operate this punch, if it is to have 30 in. diameter of center of gravity of rim, make 175 R. P. M., and the velocity fall off 20% during the actual punching operation.

3. Sailors find their latitude at sea by observing the altitude of the North Star. Prove that the latitude of the observer is measured by the altitude (angular distance above the horizon) of the North Star. (Note: For practical purposes, the direction of the North Star from the observer is considered parallel to the axis of the earth.)



4. To construct the pattern for cutting out sheet metal in the construction of elbows with any given angle ABC and any given diameter *d*. The figure suggests the construction. Construct such a pattern.



*By J. V. Collins, State Normal School, Stevens Point, Wis.*

5. How many gallons each of cream containing 30% fat and milk containing 5% fat shall be mixed so as to produce 10 gallons of cream containing 25% fat?

8. If a day's diet should consist of 4.5 oz. proteid, 2 oz. fat, and 18 oz. carbohydrate, and be made of whole wheat cereal, milk, and eggs, how many oz. of each are needed? Per cents of ingredients as given in U. S. Dept. of Agriculture are cereal, 14% proteid, 2% fat, 72% carbohydrate; milk, 3% p., 4% f., 5% c.; eggs, 15% p., 10% f.

From problems 5 and 6 we see that (1) certain unknown quantities of *two* ingredients are mixed, each containing certain known per cents of *two* substances, so that the result shall contain a given amount of each substance; (2) certain unknown quantities of *three* ingredients are mixed, each containing certain per cents of *three* substances, so that the mixture shall contain a given amount of each substance. Problems occur involving *two* ingredients and *three* substances. Thus Portland cement contains 65% lime, 25% silica, 10% aluminum. If only limestone, rock, and clay are used and the clay has not the proper proportions of silica and aluminum, the problem is impossible, but if three ingredients are used, as two kinds of clay and one of limestone, or one kind of clay and two kinds of limestone, certain proportions of each ingredient may give the desired proportion of the substances. In the case of powder, composed of saltpeter, charcoal, and sulphur, in which each ingredient is pure, the problem of finding how much by weight of each is needed is arithmetical, not algebraic.

7. How many quarts of water must be mixed with 250 quarts of alcohol 80% pure to make a mixture 75% pure?

8. Pantograph. (a) If the two tracers and the vertex of the pantograph are in a straight line to begin with, will they continue in a straight lie when the apparatus is moved? Prove the answer.

(b) If one tracer follows a straight line, prove that the other tracer also follows a straight line.

(c) Prove that if one tracer follows the perimeter of a polygon, the other traces a similar polygon.

A drawing of a pantograph tracing the polygons should accompany exercises.

*By Herbert E. Cobb, Lewis Institute, Chicago.*

Work is done when resistance is overcome. It is measured by the product of the force into the distance over which the force acts. $W=f\times s$..

9. A steam crane lifts a block of granite weighing 2 tons 80 feet. Find the work done.

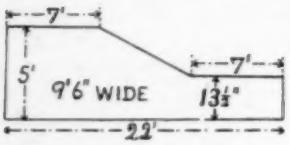
10. The tractive force of a locomotive is 12 tons. Find the work done in hauling a train 2 miles.

11. A pump is raising 2,000 gallons of water per hour from the bottom of a mine 400 ft. deep. How many foot-pounds of work are done in 3 hours, a gallon of water weighing 8.3 lbs.?

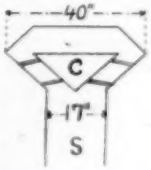
12. The plunger of a force-pump is 4 in. in diameter, the length of the stroke is 3 ft., and the pressure of the water is 36 lbs. per sq. in. Find the work done in one stroke.

13. A horse attached to a capstan bar 12 ft. long does 75,000 foot-pounds of work in going around the circle 100 times. Find the pull in pounds.

*By Mr. Hagenow, Armour Institute, Chicago.*




14. Find the capacity in gallons of a water tank for a locomotive tender, the dimensions being shown in the figure.




15. In a smoke stack for a locomotive, find the diameter of the base of cone C so that the annular space may be equivalent to the opening of the cylindrical part of the stack S.

*By C. I. Palmer, Armour Institute of Technology, Chicago.*

16. Portland cement is built on the following formula: (per cent of silica$\times$2.8)+(per cent of alumina$\times$1.1)=per cent of lime. It is desired to make Portland cement of a clay having 12.5% lime, 44.5% silica, and 10.1% alumina; and a marl having 51.2% lime, 4.1% silica, and 0.66% alumina. Find how many parts of the marl should be mixed with 100 parts of the clay.

17. A pulley 14″ in diameter on a driving shaft making 192 revolutions per minute is belted to a pulley 8″ in diameter. Find the diameter of a pulley on the same shaft with the 8″ pulley that when belted to a 6″ pulley on a machine will give it 1,400 revolutions per minute.

18. How many cubic yards of concrete in a battered retaining wall 15′ high, 2′ thick at the top, and 5′ thick at the bottom, and 50′ long?

19. A tapered stack, having a flue 10′ in diameter, is made of brick; it is 175′ high; and the wall is 4′ thick at the base and 1′ 6″ thick at the top. Find the weight of the stack, if one cubic foot of brick weighs 112 lb.

20. Find the diameter of a down-spout for a roof 60′ by 120′, using 1 sq. in. of pipe area for every 175 sq. ft. of roof area.

21. Find the diameter of a cast iron bearing plate for a column load of 40,000 pounds on a concrete pier, allowing 200 pounds per sq. in. as the safe strength of concrete.

22. Find the weight of a cast iron column 10′ long, 10″ in diameter, and 1″ thick, allowing 450 pounds per cubic foot of cast iron.

NOTE.—The above problems are printed just as received from the contributors, without any attempt by the Committee to rearrange them or to classify them into *types*. Evidently they might be classified according to the *fields of application* in which they occur, or according to the *conventional division into algebra and geometry* or into special *types within these subjects depending upon their nature and methods of solution*. (See article by Committee in March number of SCHOOL SCIENCE AND MATHEMATICS.) The problems here given will be suggestive to teachers in search of other real applied problems. Teachers may collect other similar problems in the same general fields which these problems suggest, with different data, or similar problems in entirely different fields.

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## COMMUNICATION.

Three years ago an English firm introduced the "Nivoc" laboratory stencil, which in this laboratory has proved itself so useful that it seems well to bring it to the attention of chemistry teachers more generally. It consists of a transparent sheet of celluloid 55x145 mm. from which has been cut in convenient size the commonest forms of apparatus used in elementary chemistry. The student, in writing up his notes, can by the aid of the stencil make drawings with a degree of neatness and accuracy hitherto impossible and all in much less time than it would take for a poor drawing. We have become so convinced of the value of these stencils that they are required of the student in elementary chemistry in these laboratories.

This stencil can now be obtained in this country from the L. E. Knott Apparatus Company of Boston under the name of the "Harcourt" stencil, at the price of twenty-five cents.

*Syracuse University.* H. MONMOUTH SMITH.

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In connection with textile machinery, there is a large field of application for aluminum in the making of moving parts, such as rollers, bobbins, etc., for besides the large saving in cost at the present low price of the metal, there is also a large saving in weight, resulting in considerable economy in driving power.

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That the *a* particle from radio-active substances is an atom of *helium* carrying a positive charge, equal to twice the charge of the hydrogen atom, is conclusively proven according to Rutherford & Royds. See *Chemical News*, January 29, 1909.

## CEMENT AS A SUBSTITUTE FOR WOOD.

### Probable Effect on Lumber Prices.

The relation between the increasing use of cement and the diminishing timber supply in the United States has been the subject of some interesting correspondence between the Geological Survey and the Forest Service at Washington. In a letter to the Forester, the Director of the Survey took occasion to quote from a statement of a large Philadelphia firm to the effect that it would be difficult to estimate what the additional drain on the lumber supply would have been during the last few years had not cement come into such general use. The Forester replied in part as follows:

"The Forest Service is watching with a great deal of interest the increasing use of cement and other substitutes for wood. They are undoubtedly having some influence on the price of lumber, though I do not think that up to the present time they have greatly retarded the advance in lumber prices. The fact is that our industrial progress has been so great that our requirements for every kind of structural material have increased tremendously. We are using at the present time more lumber per capita than ever before, and probably twice as much per capita as we did fifty years ago. The conclusion can not be escaped, therefore, that in the future we must depend more than in the past on other materials than wood for certain purposes at least. As to the increase that will take place in the production of cement, my impression is that this will be very great."

If the increase in the use of cement in the United States in past years is to be regarded as any index to its future use, the conclusions of the Forester are well founded. The statistics of the production of minerals show that our output of cement has more than doubled in the last five years, and it is well known that its use is being very widely extended. This is due to two conditions: In the first place, excellent cement materials are common in almost all sections of the country; in the second place, reinforced concrete for heavy building material is receiving increased favor among engineers, while in the country regions large amounts of cement are being used for building blocks for smaller structures. Reports received by the Survey during the six years from 1902 to 1907 show that the production of cement in the United States has increased from 25,000,000 barrels, valued at approximately $25,000,000, to 51,000,000 barrels, valued at $55,000,000, the annual statistics showing a steady increase in production with some slight fluctuations in price.—*U. S. Geol. Survey Bulletin.*

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The profession of mining engineering enjoys a peculiar distinction; it relates to a business which is fundamental in the progress of the world. The only original sources of wealth are the products of agriculture and mining. All else is man-made; these two are fundamental, pristine, the really indispensable necessaries of the human race.

## DECAY IN WOOD PREVENTED.

MONEY SAVED AND LESS MATERIAL NEEDED.

It is estimated that a fence post, which under ordinary circumstances will last for perhaps two years, will, if given preservative treatment costing about 10 cents, last eighteen years. The service of other timbers, such as railroad ties, telephone poles, and mine props, can be doubled and often trebled by inexpensive preservative treatment. To-day, when the cost of wood is a big item to every farmer, every stockman, every railroad manager—to everyone, in fact, who must use timber where it is likely to decay—this is a fact which shou'd be carefully considered.

It is easy to see that if the length of time timbers can be used is doubled, only half as much timber will be required as before, and only one half as much money will need to be spent in the purchase of timber. Moreover, many woods which were for a long time considered almost worthless can be treated and made to last as long as the scarcer and more expensive kinds.

Of the actual saving in dollars and cents through preservative treatment, a fence post such as was mentioned at the beginning might serve as one example. The post is of loblolly pine, and costs, untreated, about 8 cents, or, including the cost of setting, 14 cents. It lasts about two years. Compounding interest at 5 per cent, the annual charge of such a post is 7.53 cents; that is, it costs 7.53 cents a year to keep the post in service. Preservative treatment costing 10 cents will increase its length of life to about eighteen years. In this case the total cost of the post, set, is 24 cents, which, compounded at 5 per cent, gives an annual charge of 2.04 cents. Thus the saving due to treatment is 5.49 cents a year. Assuming that there are 200 posts per mile, there is a saving each year for every mile of fence of a sum equivalent to the interest on $219.60.

In the same way preservative treatment will increase the length of life of a loblolly pine railroad tie from five years to twelve years and will reduce the annual charge from 11.52 cents to 9.48 cents, which amounts to a saving of $58.75 per mile.

It is estimated that 150,000 acres are required each year to grow timber for the anthracite coal mines alone. The average life of an untreated mine prop is not more than three years. By proper preservative treatment it can be prolonged by many times this figure. Telephone and telegraph poles, which in ten or twelve years, or even less, decay so badly at the ground line that they have to be removed, can, by a simple treatment of their butts, be made to last twenty or twenty-five years. Sap shingles, which are almost valueless in their natural state, can easily be treated and made to outlast even painted shingles of the most decay-resistant woods. Thousands of dollars are lost every year by the so-called "bluing" of freshly sawed sapwood lumber. This can be prevented by proper treatment, and at a cost so small as to put it within the reach of the smallest operator.

In the South the cheap and abundant loblolly pine, one of the easiest of all woods to treat, can by proper preparation be made to take the place of the high-grade longleaf pine for many purposes. Black and tupelo gums and other little-used woods have a new and increasing importance because of the possibility of preserving them from decay at small cost. In the Northeastern and Lake States are tamarack, hemlock, beech, birch, and maple, and the red and black oaks, all of which by proper treatment may help to replace the fast-diminishing white oak and cedar. In the States of the Mississippi Valley the pressing fence-post problem may be greatly relieved by treating such species as cottonwood, willow, and hackberry.

Circular 139 of the Forest Service, "A Primer of Wood Preservation," tells in simple terms what decay is and how it can be retarded, describes briefly certain preservatives and processes, gives examples of the saving in dollars and cents, and tells what wood preservation can do in the future. The circular can be had free upon application to the Forester, Forest Service, Washington, D. C.—*Forest Service.*

## SUGGESTIONS TO WOODLOT OWNERS.

The woodlot is a very large factor in the production of the raw material which supports the fourth greatest industry of the country. Although the area of a single woodlot is small, the cuttings are more frequent than from the large timber tract, and it is probable that the total amount of timber produced by woodlots is greater than that produced by timber tracts.

Not only this: a farm without a good woodlot is incomplete. Usually, not less than one-eighth of the acreage of every farm should be devoted exclusively to timber growing. If properly managed the woodlot will supply the farmer with posts, fuel, and building material, as suits his convenience best. It may even furnish some timber for market.

Again, the farm house should be protected from winds. For the live-stock grower, shielding the barnyard and feedlots will reduce the quantity of grain necessary to fatten stock, since less food will be required to keep up the animal heat in winter. A belt of forest trees will greatly reduce the danger of late frosts to the fruit blossoms of an orchard.

The woodlot should occupy the waste land not suitable for farm crops. Steep hillsides, ravines, swamps, sand dunes, creek banks, rocky slopes, and corners cut off by ditches, creeks, or railroads will sustain a good growth of timber and become an important source of revenue. Forest growth on steep slopes and river banks protects them from erosion by heavy rains and freshets.

The woodlot affords one of the best opportunities for the practice of forestry. It is accessible enough to allow of moderate cuttings at frequent intervals; its protection from trespass, grazing, and fire does not require an elaborate scheme of defense; and taxation is not so great a burden because the revenue in farm supplies more than meets this item every year.

In Circular 138, recently issued by the Forest Service and which can be had free of the Forester at Washington, the model woodlot and the present condition of the woodlots of the Ohio Valley region are discussed. Suggestions are given as to uses of the woodlot and its protection from grazing, fire, and wind, and from injury during cutting, and for the improvement of typical woodlots of dense first growths, of mature open stand, of dense young stand, and sapling thicket.—*Forest Service.*

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## THE CHARGE AND NATURE OF THE *a* PARTICLE.

In recent experiments by Rutherford and Geiger, they arrived at the following conclusion. The atomic weight of an *a* particle is 3.84. The atomic weight of the helium atom is 3.96. Taking into account probable experimental errors, they conclude that the *a* particle, after it has lost its positive charge, is a helium atom.

"There is direct evidence in the case of radium that each of the *a* ray changes is accompanied by the expulsion of one *a* particle from each atom. Consequently since the atomic weight of radium is 226 the atomic weight of the emanation is 222, and of radium A 218."

"One atom of radium, in breaking up, emits one *a* particle and gives rise to one atom of emanation of atomic mass 222. Since $3.4\times10^{10}$ *a* particles are expelled per second per gram of radium, the number of atoms of emanation produced per second is the same. There are $2.27\times10^{19}$ molecules in 1 c.c. of any gas at standard pressure and temperature. The volume of the emanation produced per second per gram is $1.25\times10^{-9}$ c.c. The maximum volume of the emanation from 1 gram of radium is therefore 0.585 cu. mm. The volume of helium produced per gram is $5.0\times10^{-9}$ c.c. per second, which is equal to 0.43 cu. mm. per day.

The life of radium works out to be 1760 years. Heating effect per gram of radium, 113 gram-calories per hour. Number of atoms of radium breaking up per second, $3.4\times10^{10}$.

Hon. J. R. Strutt, as the result of studies on various rocks, arrives at these conclusions:

1. Phosphatic nodules and phosphatized bones of all geological ages possess marked radio-activity many times higher than that of rocks. This activity is due to products of the uranium series.
2. Helium has been detected in these materials.
3. The ratio of helium to uranium oxide has been measured. This ratio does not strictly follow the order of super-position of the strata, but high ratios are not met with in the younger deposits, whereas they are common in the older ones.—*Abstracted from Proceedings of the Royal Society.*

## THE SCHOOL ROOM DUST PROBLEM.

One of the most unpleasant things with which teachers in public schools have to contend is the method of sweeping and cleaning the school rooms. In most cases the rooms are dry swept, which means that the dust and dirt is made the vehicle for the distribution of many pathogenic germs. This dust evil is one of the greatest problems of sanitation left for public school authorities to solve. Much attention has been given, as it should be, to the proper heating and ventilating of our school buildings. The food and water supply have received much attention. It is seen that the plumbing is properly set and in many schools fire escapes have been provided. These things are all good, and it is well that they are being looked after with much care and intelligence. This dust problem, in many respects the most vital of them all, has received the least attention. Dry sweeping, instead of removing the dirt, divides it into finer particles which float in the air and are finally settled on everything in the room to be stirred up the next day by the pupils and taken into their lungs. The wonder is that we do not have more pupils infected with one form or another of tuberculosis.

The eradicating of the dust nuisance lies in the installation of a vacuum cleaning apparatus in every school, and, for that matter, in every public building. In new buildings the extra cost would not be very much, as the exhaust pipes would be placed in walls during construction, all leading to the exhauster in the heating room. The extra expense would soon be met by the saving in cleaning materials and janitor service. The crayon dust nuisance may also be overcome by using vacuum blackboard erasers, which are the very latest things in school building equipment.

Mr. Robert L. Cooley, principal of the Sixteenth District School No. 1 of Milwaukee, Wis., has had installed in his building a vacuum cleaning apparatus, and his testimony is nothing but praise for its successful working in his school. He says the system has completely solved the dust nuisance with him.

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Sulphate of lime and sulphate of magnesia remain in solution until a temperature of from 300° to 360° F. is reached. In other words, they pass on into the boilers, even though the water be heated previously and filtered, and with the concentration from the boiling off of the water, in time form a hard, porcelain-like scale, difficult to dislodge, and even more dangerous than the soft scale from the carbonates.

Apart from the demand for manganese from the steel trade, there is a limited consumption in the preparation of bleaching powder, of oxygen and of disinfectants. It is further used as a decolorizer in glass manufacture, and also as a coloring matter; but, though for these purposes a specially pure ore of particular composition is demanded and an enhanced price paid, the consumption is relatively so small that it does not affect the manganese industry, so that the demand, and consequently the price, therefore, fluctuates directly with the needs of the steel trade.

## ARTICLES IN CURRENT MAGAZINES.

*Conservation* for February: "The Twenty-eighth Annual Meeting"; "The Joint Conservation Conference" (Concluded from January); "The Woman's National Rivers and Harbors Congress," Lydia Adams-Williams: "The Appalachian-White Mountain Hearing"; "Hope for Forestry Legislation"; "The Destruction of American Forests," Richard H. Donai Boerker.

*Photo-Era* for March: "The Case against the Moving-Picture," C. H. Claudy; "Sulphur and Permanence," R. Child Bayley; "On the Comparative Merits of Different Developers—I. Materials and the Criteria of Perfection," Malcolm D. Miller; "Vigorous Carbon Transparencies from Weak Negatives"; "A Formula for Tank Development," R. E. Crane; "Latitude in Bromide Work," C. Winthrope Somerville.

*School Review* for March: "Relation of Industrial to General Education," Charles De Garmo; "The Teaching of Agriculture in the High School," F. M. Giles; "A Study in Adult Education," Frank A. Manny; "Commercialism: The Educator's Bugbear," Lewis M. Terman.

*School World* for March: "Experimental Psychology and Education," J. A. Green; "The Teaching of Algebra, Its Connection with Arithmetic," P. B. Ballard; "Continuation Schools in Switzerland."

*Scientific American Supplement* for February 20: "The Cavern of Proumeyssac," illustrated, E. A. Martel. For February 27: "Darwinism Fifty Years After," David Starr Jordan: "Electroculture of Plants": "The Life History of the Termite," K. Escherick; "The Production of Ozone," Arthur W. Ewell; "Earthquake Forecasts," G. K. Gilbert. For March 6: "The Rotaplane"; "Recent Developments in Galvanizing"; "The Aniline Dyes," Max Ohlman; "The Internally-Fired Helical Furnace," C. M. Ripley; "The Dynamics of Life," A. Drzewina.

*Monthly Weather Review* for February: "Deficient Humidity Indoors," F. H. Day; "Wireless Telegraphy in the Service of Modern Meteorology," P. Polis; "Remarkable Snow Storms at Grand Haven, Michigan," C. H. Eshleman.

*Popular Science Monthly* for March: "The Electric Operation of Steam Railways," J. B. Whitehead; "The Influence of Radium Rays on a Few Life Processes of Plants," C. Stuart Gager; "The Work of Boards of Health," Dr. George A. Soper; "A Biographical History of Botany at St. Louis, Missouri," Dr. Perley Spaulding; "Fire's Havoc a Senseless Waste," F. W. Fitzpatrick; "The World's Annual Metal Crop," Theo. F. Van Wagenen; "Science and Morality," Albert P. Mathews; "The American Public School," James P. Munroe.

*Review of Reviews* for March: "Into Africa with Roosevelt," Edward B. Clark, with portraits; "Improved Methods of Fruit-Handling," Francis John Dyer, with illustrations; "Colonizing the Tramp," Gustavus Myers, with illustrations; "The New Union Among the States," W. J. McGee, with illustration; "Africa in Transformation," Cyrus C. Adams, with map; "The Native Problem in South Africa," Olive Schreiner; "Bishop Hartzell and His Work in Africa," Ferdinand Cowle Iglehart, with portrait; "Safety of Travel on the Modern Ocean Liner," E. A. Stevens, with illustrations; "The Motor Car and Its Owner," E. Ralph Estep, with illustrations; "The Epoch of Roosevelt," Judson C. Welliver.

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The firm of Bausch & Lomb has suffered a severe loss in the death of its Second Vice-President, Mr. Henry Bausch. The death of Mr. Henry Bausch occurred recently in Augusta, Ga., where he had gone with his family in order to recover his health. Mr. Bausch has done much during the last few years in forwarding the rapid development of the firm of Bausch & Lomb. It was largely through his efforts that many of the new devices of an optical nature have been put on the market. He was well versed in the science of optics, and many of the optical instruments manufactured by this firm are creations of his brain. He secured his education at Cornell University where he was well known by the Editor of this Journal. The loss to the firm is almost irreparable.

## THE KANSAS ASSOCIATION OF MATHEMATICS TEACHERS.

The Kansas Association of Mathematics Teachers held its sixth annual meeting in Topeka, December 30 and 31, the President, Professor W. A. Harshbarger of Washburn College, presiding. The following subjects were discussed:

"The Extent and Character of the History of Mathematics that can Profitably be Taught in the High School," was discussed by Professor H. B. Newson of the University of Kansas. Professor Newson recommends that where the histories are not easily accessible to the pupils, the teacher give in an informal way a sort of history-story, connecting dates and events of interesting problems with the history work or with the English.

Professor A. B. Frizell of Midland College, Atchison, spoke on the subject, "What is an Area?' in which he considered some of the inherent defects in the current definitions of area; and proposed to substitute the ideas of approximation by excess and defect, which, together with the estimation of error of approximation, form the core of the process used in common by the engineer and the mathematician.

Dr. Paul Wernicke of the University of Kansas gave a paper on "Secondary Mathematical Instruction in Germany, and Its Proposed Reform," in which he showed the progress of Mathematical instruction in the three types of secondary schools, explaining that modern methods, models, plotting, physical and mechanical applications, were introduced from time to time, mainly on the initiative of competent teachers, whose work has found recognition in the "Plans of Instruction" laid down by the government for the guidance of these schools. Calculus, which involved the notion of a "limit" too hard for the youthful mind, was formerly precluded from the secondary schools. The leaders of a movement for further reform, foremost among them Felix Klein of the University of Goettingen, contend that Calculus is necessary to the progress of modern science and should be given in the higher classes of the nine-year curricula of secondary schools.

Miss Emma Hyde of Emporia High School discussed "Correlation of Algebra and Geometry in the High School," advocating Unified Mathematics as reported by the committee selected by the Central Association. One very interesting feature was the Question Box in charge of Mr. Wagner, Emporia High School. It was considered such a success that it will be continued next year.

The Nominating Committee reported as follows: For President, Miss Effie Graham, Topeka High School; for Secretary-Treasurer, Emma Hyde, Emporia High School. EMMA HYDE, Secretary.

## EASTERN ASSOCIATION OF PHYSICS TEACHERS.

[Abstracted from the Proceedings.]

The fifty-second meeting of this association was held, January 23, in the Franklin Union, Boston, the meeting being called to order by the president, Mr. Fred R. Miller. The secretary's printed report of the last meeting was accepted.

Mr. J. B. Merrill, from the committee on magazine literature, presented a report covering the period from October to December, 1908, inclusive. Mr. R. C. Chittenden presented the report of the committee on current events.

Mr. Irving O. Palmer, for the committee on relations with other organizations, reports that in the judgment of the committee it would be unwise for this association to affiliate with the American Federation of Teachers of Mathematical and Natural Science. The report of the committee was unanimously adopted.

Mr. George A. Cowen, from the committee on new apparatus, showed a new form of linear expansion apparatus which he has designed and used for some time in the West Roxbury High School. This apparatus is designed so that water of any desired temperature, as well as steam, can be passed through a metal or glass tube firmly held at one end and allowed to move freely at the other upon an axle which it causes to revolve in a nearly frictionless bearing.

This apparatus will be described in a future number of this journal.

Mr. W. F. Rice and Mr. Calvin H. Andrews led a discussion on the question: "Is it Expedient to Teach the Absolute System of Measurement in Secondary Schools?" Mr. Rice contended that it was not, in a splendid argument, and Mr. Andrews advocated, with just as much enthusiasm, for its teaching.

Professor Louis Derr of the Massachusetts Institute of Technology addressed the association on "Modern Protographic Lenses." The paper was illustrated with photographs and lantern slides.

At the afternoon session Mr. G. A. Cowen showed a simple spiral spring device by means of which the pupil can easily verify Hook's law. He also described a neat method of jacketing a steam trap by slipping a piece of rubber tubing over it. Mr. C. M. Hall exhibited a rotating apparatus used for determining the mechanical equivalent of heat.

Professor E. H. Hall explained the principle of the Porro prism used in the Zeiss field glasses by means of which an image is obtained which is not inverted or reversed. He also showed some interesting experiments concerning the coefficient of friction between metals when they are wet and when they are dry.

Mr. Walter B. Russell, director of the Franklin Union, being introduced, gave a very interesting address on "Industrial Schools."

At the close of this address a business meeting was held. Several persons were elected to membership in the association. The annual reports of the secretary and treasurer were read and accepted.

The election of officers for the ensuing year resulted as follows: President, N. Henry Block, Roxbury; Vice-President, Frank M. Greenlow, Newport, R. I.; Secretary, Fred H. Cowan, Jamaica Plain; Treasurer, Percy S. Brayton, Medford; Executive Committee, the officers *ex-officio* and Fred R. Miller, Newton Highlands, Arthur N. Burke, Waltham, and Harver W. Le Sourd, Milton.

C. H. S.

## A SUCCESSFUL CULTURE METHOD FOR AMOEBA.

A reliable and simple culture method for amoebae, which makes it possible to grow continuously this valuable animal type for class study and research, has been discovered by A. H. Cole, of the Chicago Normal School. Thorough tests have been made with amoebae from typical habitats in cultures which have been subjected to ordinary laboratory conditions during the fall and winter. Every culture has been a success. The record of a culture made last September is, in brief, as follows: At first it contained very few amoebae, but these multiplied rapidly, were numerous early in November, early in December fifty students found the amoebae in ideal condition as to numbers, size, activity and feeding while studying them for three or four days, and two weeks later an examination of a few drops of the culture under a two thirds inch objective and one inch eyepiece showed twenty-four amoebae in the first field, i. e., in a circle .06 of an inch in diameter. Frequent tests have shown an abundance of two or more species at all times since then, and the culture, now nearly six months old, shows no signs of death. The cultures need but very little attention, and they always contain an abundance of desmids or diatoms or of both. The directions for making the cultures have been issued as a copyrighted supplement to Cole's "Manual of Biological Projection and Anesthesia of Animals," advertised on another page of this issue, and will be included in all copies hereafter sent out. Present owners of the "Manual" can obtain the supplement gratis by enclosing return postage with address to the publisher, A. H. Cole, 6022 Monroe Ave., Chicago.

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## FENCE POSTS MADE DURABLE.

### Woods Given Sixteen Years' Additional Service by Preservative Treatment.

Fence posts of many kinds of cheap woods which ordinarily would soon decay if set in the ground can be made to last for twenty years by a simple treatment with creosote. Most of the so-called "inferior" woods are well adapted to the treatment, and this is especially true of cottonwood, aspen, willow, sycamore, low-grade pines, and some of the gums. When properly treated, these woods outlast untreated cedar and oak, which are becoming too scarce and too much in demand for other uses to allow of their meeting the demand for fence posts.

Impregnation with creosote has been greatly cheapened by the introduction of the "open tank," which can be installed at a cost of from $30 to $45, or much less if an old boiler is used. A tank with a bottom 12 square feet in area will suffice for treating 40 or 50 6-inch posts a day, or double this number when two runs per day can be made. The absorption of creosote per post is about as follows: Eucalyptus, one-tenth gallon; willow, two-tenths gallon; sassafras, ash, hickory, red oak, water oak, elm, and maple, four-tenths gallon; Douglas fir, quaking aspen, and black walnut, six-tenths gallon; sycamore, cotton-

wood, and lodgepole pine, seven-tenths gallon. The price of creosote is about 10 cents per gallon in the East and Middle West, 16 cents per gallon on the Pacific coast, and 27 cents per gallon in the Rocky Mountain States. The cost of treating a post will therefore vary from 4 to 15 cents. Properly treated, it should give service for at least twenty years.

Experiments of the Forest Service show that with preservative treatment the durability of lodgepole pine in Idaho is increased sixteen years. The cost of creosote is there relatively high, yet by treating posts there is a saving, with interest at 6 per cent, of 2 cents per post yearly. More important than the saving, however, is the fact that through preservative treatment other woods are fitted to take the place of cedar, of which the supply is rapidly becoming exhausted. A detailed description of experiments in preserving fence posts, together with practical suggestions for treating them on a commercial scale, are contained in Circular 117 of the Forest Service. This publication can be obtained upon application to the Forester at Washington.—*Forest Service.*

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The quantity of coal consumed in the manufacture of coke in 1907 was 61,946,109 short tons, valued at $72,784,851. The value of the coke produced from this coal was $11,539,126, a difference of $38,754,275, which represents the profits on the coking operations less the cost of manufacturing and the expenses of administration and selling. In 1906 the value of the coal used was $62,232,524, and the value of the coke produced was $91,608,034, a difference to cover all expenses of manufacture, administration, and profits of $29,375,510.

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Consul A. G. Seyfert of Owen Sound, Ontario, furnishes the following statistics concerning cement in Canada:

One-half of all the cement manufactured in Canada is manufactured in the locality of Owen Sound, the daily output here being 3,000 barrels. The demand for concrete material for building the transcontinental railroad has increased the output to the full capacity of the plants during the past year. During 1907 Canada produced 2,500,000 barrels of Portland cement, and imported 1,000,000 more, nearly all from the United States.

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Where gold in ore does not readily amalgamate, this is, in most instances, due to the fact that the gold is coated with some foreign substance. This coating may be iron oxide, silica, calcium carbonate, or any other of a number of substances.

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A large consignment of rock from the Premier Tin Co., Zululand, has been brought to Durban, and the percentage of tin is such as is likely to insure a profitable mine.

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Large quantities of specially pure aluminum tubes are being used for various purposes in connection with acid work, under which condition the pure metal remains practically unaffected.

## BOOK REVIEWS.

*Practical Arithmetic, by A. Consterdine and S. O. Andrew.* 2s. Pp. Part I, 138, Part II, 96. John Murray, London. 1907.

Teachers of arithmetic will find much of interest in this book. It is very much like the Practical Mathematics, and is indeed a companion volume to that book for the use of day scholars between the ages of nine and twelve. H. E. C.

*Practical Arithmetic Examples, compiled by J. L. Martin.* Part I, pp. 124, 6d. Part II, pp. 124, 6d. John Murray, London. 1908.

These little pamphlets contain many excellent practical problems in decimals, common fractions, measurements, areas, ratio, proportion, graphs, interest, percentage, and volume. The out of the ordinary problems would be sure to arouse the interest of a class in arithmetic. H. E. C.

*The Parallel Course Drawing Books, by C. S. & A. G. Hammock. Four books of forty-one pages each.* D. C. Heath & Company, Boston.

This set of drawing books has undoubtedly been gotten out in response to a demand for a set of books presenting the pencil as the medium of drawing. The several lessons in the books are planned to be executed both with pencil and brush. The instructions accompanying each exercise are complete, and a pupil should be able to make a very acceptable reproduction of the drawings with but very little help from the teacher. Each book contains a set of instructions to the teacher as well as instruction more particularly to the pupil who is also told how to mix colors. The books will undoubtedly fill a want which has long been felt by teachers of drawing in our public schools. Any School Board will not be making a mistake if they adopt this set of books for their drawing course. C. H. S.

*The Romance of Modern Mechanism, by Archibald Williams.* Crown, 8 vo., pages xi+356. 30 illustrations. $1.50 net. The J. B. Lippincott Co., Philadelphia.

The title of this book makes us wonder whether we know just what "romance" means, for "mechanism" and "machinery" appeal to the average reader as being too prosaic to furnish proper material for anything in the line of romance. That, however, is his feeling before he reads Mr. Williams' book. Before he has read far he finds what the Century Dictionary tells us is romance—"a blending of the heroic, the marvelous, the mysterious, and the imaginative, etc."

It would be instructive to print here the titles of the twenty-seven chapters each devoted to some particular phase of the romance, but that is, of course, impossible. Some of the titles cannot be omitted; for instance, chapter III is on workshop machinery—the lathe—planing machines—the steam hammer—hydraulic tools. Probably comparatively few people know that armor plate is now made under a hydraulic press, giving a pressure of 12 tons per square inch and compressing $32^{1}/_{2}$ cubic feet of billets into 31 cubic feet of finished plate. Ordinarily we regard steel as only slightly compressible.

The chapter on Internal Combustion Engines is easy reading and explains very lucidly the "Otto" cycle and also that deeper mystery the "two-cycle" motor.

Among other chapters of particular interest are those on "The Machinery of a Ship"—the extent of which few appreciate, "Dairy Machinery," "Pneumatic Mail Tubes," and, almost the strangest of all, "Sculpturing Machines," which can make four copies simultaneously from one original. F. T. J.

*Essentials of Botany, by Joseph Y. Bergen*, published by Ginn & Company, Boston, Mass. 10+380 pages. Ill. 12mo. Price, $1.20.

The botanical text-books of this author are too well known to require description, but it is safe to say that this addition to the list will be welcomed by all who have found Bergen's botanies useful in the past. In its general arrangement it differs very little from the two earlier books by the same author. The discussion of the seed plants is essentially the same as that found in the "Elements of Botany," but the portion devoted to cryptogams has been extended so as to include all the forms usually studied in secondary schools. All the necessary directions for the laboratory study of the plants described are clearly given.

An excellent chapter on plant breeding and a shorter one on the subject of forestry give the needed recognition to the practical application of the science to the economic problems of the hour. A less successful attempt has been made to discuss the history of the plant kingdom within the limits of a single chapter.

One might wish that the ecological aspect of the subject had received more emphasis and that only those details of floral structure had been included that can readily be shown to have a connection with the evolutionary advance of seed plants. There is much, however, to commend in the book as the material is well selected, logically arranged, and clearly discussed; the amount being quite sufficient for a year's course in any secondary school. The volume is well illustrated, several of the new figures and diagrams being especially good.

GEO. D. FULLER.

*Plane and Solid Geometry, by Elmer A. Lyman, Professor of Mathematics, Michigan State Normal College.* 12 mo., pp. 340. $1.25. American Book Co.

In Professor Lyman's book an attempt has been made "to prepare a geometry through which it will not only be possible, but necessary, for the student to work his way, relying on his own reasoning powers." The effort has been in good degree successful.

The scope of the work is about the same as that of the greater number of standard geometries, although a considerable number of propositions that have appeared in text-books from the days of Davies and Loomis have been inserted as exercises, or wholly omitted. The wisdom of this change can be determined only by the actual test of the classroom. Another change, one somewhat more radical and regarding which there will doubtless be a marked difference of opinion, although it is in accordance with present tendencies, consists in a very summary disposal of the incommensurable case.

The proofs are in the main clear, and such as to inculcate in the student the habit of self reliance. Well chosen exercises are inserted

after many of the propositions upon which they are dependent, and longer lists are found at the end of each book. Altogether there are more than a thousand exercises, some of which afford interesting practical applications. A few historical notes add life and interest to the subject, and will doubtless be fruitful in arousing inquiry in the mind of the thoughtful student.

From the standpoint of the many teachers who are coming to believe in the teaching of mathematics, rather than the discrete branches of that subject, to the pupil from the beginning of his career, it is to be regretted that there are not many easy applications of algebra made almost from the beginning of the first book. Perhaps, also, much more in the way of numerical computation might profitably have been carried along with the study of the pure logic. The book as a whole is a teachable one, and it is a model of typographical excellence as are all that come from the press of the American Book Company.

DUANE STUDLEY.

*Arithmetical Abilities and Some Factors Determining Them, by Cliff Winfield Stone, Ph.D.* Pp. 101. Postpaid, $1.00. Published by Teachers College, Columbia University, New York City. 1908.

In the investigations for this research thesis the author has attempted to discover the relation between distinctive educational procedures and the resulting products. The parts of the larger question to which he has given especial attention are: (1) What is the nature of the product of the first six years of arithmetic work? (2) What is the relation between distinctive procedures in arithmetic work and the resulting abilities? Tests to determine the ability of sixth grade pupils were conducted personally by the author in twenty-six school systems in New England, the Middle East, and the Middle West, and the courses of study in these school systems were rated by twenty-one judges. The test consisted of twelve minutes' work in addition, subtraction, multiplication, and division, followed by fifteen minutes' work on problems in reasoning.

In Part II the results of the tests are tabulated and studied to determine the nature of the product of the first six years of arithmetic work. The questions considered are: (1) Is the net result of the arithmetic work of the first six years *a product*, or is it *several products?* (2) To what extent are these products uniform (a) among systems, (b) among individual pupils in the systems, (c) among boys, (d) among girls? (3) In how far does the possession of one ability imply the possession of others? The results of the tests are given in eighteen tables which are the basis of the author's conclusions, of which a few may be briefly mentioned. The net result is *several products* rather than *a product*. There is a decided lack of uniformity among the systems; the variability among individuals within a system is even greater than that among systems; the variability among boys does not appreciably differ from that among girls; ability in any fundamental except addition implies nearly the same ability in other fundamentals in both systems and individuals.

Part III deals with arithmetical abilities and time expenditure. A few of the conclusions are: In so far as these twenty-six systems are a representative measure, there is very little relation between arithmetical abilities and time expenditure in present practice; many systems are wasting time on arithmetic; in that all the coefficients that include home study are higher than those without may indicate that home study is a factor in determining abilities.

In Part IV the problem considered is: What is the relation between arithmetical abilities and the course of study in arithmetic? "The situation seems to be that the course of study is not at present the factor that it ought to be in producing abilities. In certain systems it is evidently working well, but in others there is a wide disparagement between excellence in abilities and excellence in course of study. The best courses of study will continue to be made in accord with the *social aim* and *functional psychology;* the next step in improvement is to so arrange the subject matter as to indicate how it may be so taught as to afford sufficient discipline of the right kind."

This book should be in the hands of every one who has anything to do with arranging courses of study in arithmetic, and every teacher of arithmetic could doubtless do better work if this book were studied and its suggestions heeded. It is to be hoped that similar investigations in algebra and geometry will be undertaken at the Teachers College. H. E. C.

*A Laboratory Course in Plant Physiology. W. F. Ganong. Smith College, Northampton, Mass.* Published by Henry Holt & Company, New York. 1908.

"A Laboratory Course in Plant Physiology" is a second and enlarged edition of Professor Ganong's former Plant Physiology. The book is the outcome of first-hand experience by one of the foremost teachers of botany in the United States, and as such will be welcomed by all who are interested either in research or in teaching the subject. The book does not pretend to be "a compendium of physiological knowledge," though it is filled with it, and is "designed as a contribution to educational economy," in which ambition it most certainly succeeds. The first section (56 pages) is given to a consideration of the "Educational Place, Methods of Study, and Necessary Equipment of Plant Physiology." The discussion in this section is so clear as to enable a good many teachers to define their own purposes, which they may have more or less vaguely felt. There is an especially interesting photographic and text description of the Smith College greenhouse for use in plant physiology.

In Part II, the Course in Experimental Plant Physiology (188 pages), the topics usually found in text-books upon plant physiology are found, but in this connection are found descriptions of construction and use of many pieces of apparatus designed especially by Professor Ganong, which are of the utmost importance in doing the most valuable physiological work. It is safe to say that some teachers in order properly to use some physiological apparatus that they possess, need the careful descriptions given in this connection. This helpful direction is con-

tinued in the closing part upon "Manipulation and Tables Important in Plant Physiology." The book is well-nigh indispensable to any botanist who does any physiological work, and is a distinct contribution to science education as well as to work in plant physiology. O. W. C.

"Forest Trees of the Pacific Slope," by George B. Sudworth, issued by the United States Forest Service, is a valuable contribution to the literature of trees of our western regions. The book consists of 441 pages and 207 figures. Trees belonging in 29 plant families are described. The range covered is from Northwestern Alaska to Southern California. It is interesting to note that the region contains at least 14 genera of Gymnosperms with not less than 53 species.

Both horizontal and vertical range of each species is given, a fact of great importance to a field student. Extensive notes on the physiographic environment and habit of the forms described make the book much more than a field manual. O. W. C.

"A Biological Survey of Walnut Lake, Michigan," by T. L. Hankinson, published by the Michigan State Board of Geological Survey, presents in an elaborate way the results of an intensive ecological study of a limited region. The primary purpose of the work was to study the fish life of the region selected. It was hoped that the habits and needs of economic fish might be determined to the end that they may be grown in lakes not now producing edible fish in large quantities.

The "Physiography and Geology" of the region is presented by C. A. Davis, who also presents a separate chapter upon "The Flora" of the region. A chapter upon the aquatic insects of the lake with especial reference to a few species of considerable importance as fish food was written by J. G. Needham. The rest of the book, as well as a large collection of photographic illustrations, is by Mr. Hankinson.

Amongst the interesting results of the investigation the relations that are found to exist between the white fish and the oxygen supply of the deep water is extremely interesting. It has been found that at certain parts of the year the white fish live largely upon certain *Daphnia* and upon the larvæ of a species of the midge *Chironomus*. It seems that in deep water in which the white fish feeds, the *Daphnia* and *Chironomus* can thrive only if oxygen is more abundant than is usual in the lakes of the general region. Walnut Lake, as was also found by Professor Birge to be true of Trout Lake, Wis., contains more oxygen in deep water than had previously been shown to exist in certain other lakes of similar depth. The presence of this oxygen supply seems to permit the growth of *Daphnia* and larvæ of *Chironomus*, which supplies food for the fish. There is relatively little plankton in Walnut Lake and consequently not so much decay in deeper water, thus demanding less oxygen for oxidation of decomposition products.

Interesting as is this series of results, it is less important than the exemplification of persistent and exhaustive taking of evidence from every source that could give any light on the problem and finally deducing conclusions therefrom. The method used here is evidence that more good work may be expected from the same source. O. W. C.